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Field 1:

Set Theory, Topology, Logic, Algebra, Discrete Mathematics, Number Theory

Graphs with maximal spread in some classes of graphs

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For a simple graph G, the graph's spread $s(G)$ is defined as the difference between the largest eigenvalue and the least eigenvalue of the graph's adjacency matrix, i.e. $s(G) = \rho(G) - \lambda(G)$. Characterising the graph with maximal spread is still a difficult problem. If we restrict the discussion to some classes of connected graphs of a prescribed order and size, we can determine the graphs whose spread is maximal among graphs of certain classes. In this paper, we present a unique graph whose spread is maximal in the class of bicyclic graphs with n vertices. We, also, characterise a unique cactus whose spread is maximal in the class $C(n, k)$ of cacti with n vertices and k cycles. We prove that the obtained graph is a bundle of a special form.

Some spectral properties of the graphs with (k, t)-regular sets

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Lert G be a simple graph. A (k, τ)-regular set S is a subset of the vertices of a graph G, inducing a k-regular subgraph such that every vertex not in the subset has τ neighbors in it. We consider the signless Laplacian spectra of graphs with (k, τ)-regular sets. We give several lower bounds on the index of such graphs as well as some upper bounds on the sum of the squares of the entries of the principal eigenvector corresponding to the vertices in S.

Algebraic frustration of signed graphs

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A signed graph Γ is a pair (G, σ), where $G = (V(G), E(G))$ is a simple graph and $\sigma : E(G) \to \{1, -1\}$ is the sign function on the edges of G. A cycle in Γ is said to be positive if and only if the product of its edge signs is positive. A signed graph Γ is balanced if all of its cycles are positive. If Γ is not balanced, then a suitable deletion of some vertices or edges leads to a balanced graph. Let $\nu(\Gamma)$ (resp. $\epsilon(\Gamma)$) be the minimum number of vertices (resp. edges) to be deleted such that the obtained signed graph is balanced. The values $\nu(\Gamma)$ and $\epsilon(\Gamma)$ are called the frustration number and frustration index, respectively. Let $D(G)$ be the diagonal matrix of vertex degrees, and $A(\Gamma) = (a_{ij})$ be the adjacency matrix of Γ, where $a_{ij} = \sigma(ij)$ whenever $ij \in E(G)$ and $a_{ij} = 0$ otherwise. For any signed graph $\Gamma = (G, \sigma)$, the matrix $L(\Gamma) = D(G) - A(\Gamma)$ is the Laplacian of Γ, and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$ are its eigenvalues.It is well-known that if Γ is a connected balanced graph then $\lambda_n(\Gamma) = 0$. Here we show that $\lambda_n(\Gamma) \leq \nu(\Gamma) \leq \epsilon(\Gamma)$. Hence we refer to $\lambda_n(\Gamma)$ as the algebraic frustration. Further, we study the case $\lambda_n(\Gamma) = \nu(\Gamma)$, and we give necessary and sufficient conditions which lead to the latter equality. Finally, we give some families of almost complete signed graphs for which the above equality holds and we compute their spectra.

Polygonal Numbers and Fermat's Last Theorem

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We begin with the elementary Diophantine x2 + y2 = z2 in positive integers, which we know has infinite solutions.Fermat's Last Theorem does not let us generalize this for higher powers. But we can generalize this for polygonal numbers, we can in fact prove that there are infinitely many n-gonal numbers which can be represented as a sum of m n-gonal numbers, for all m and n. Now, if we consider the above Diophantine for higher dimensional regular convex polytope numbers(squares above being two dimensional regular convex polytopes), we notice that there are special cases in each dimension where the solutions do not exist. As we see where the solutions exist and where they do not, we gain some new insights into Fermat's Last Theorem. We observe that Fermat's Last Theorem does not simply give us a family of Diophantine equations having no positive integer solutions, but rather the boundary in each dimension from where the solutions cease to exist. Lastly, based on our insights, we ask a few questions, which if answered, could actually explain in a dierent way why Fermat's Last Theorem holds!

On the signless Laplacian index of unicyclic and bicyclic graphs with perfect matching

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Among the unicyclic and bicyclic graphs with perfect matching the graphs with the largest signless Laplacian index are determined.

Some results in nodal filters in residuated lattices

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In this paper, we introduce the notion of nodal filters of residuated lattices. A filter F of residuated lattice A is said to be nodal if is comparable with any filters of A in the set of all filters of A ordered by inclusion and we study it in detail and investigate the relationships with the other types of lters and special sets in residuated lattice. Finally we obtain a characterization of the nodal filters in terms of congruences.

Topologies induced by $(3, 1, \rho)$-metric and $(3, 2, \rho)$-metrics

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For a given $(3, j, \rho)$-metric d on a set M, $j \in \{1, 2\}$, we define seven topologies on M induced by d, give some examples, examine the connection among them, and show that some other topologies induced by generalized metrics are special cases of the above ones.

Solving equations in residuated semigroups and quantales

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We inverstigate solvability of various types of equations in residuated semigroups and quantales. Most attention is devoted to Moore-Penrose equations in residuated semigroups and quantales with involution, and in particular, to Moore-Penrose equations for fuzzy relations and matrices over residuated structures of truth values (quantales, complete residuated lattices, complete Heyting algebras, etc.).

The Bruhat order for a class of binary matrices

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We discuss some results on the length of a chain in the Bruhat order for an interesting combinatorial class of binary matrices. Several open questions are discussed.

Residuation in algebras of relations and matrices

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Residuals of Boolean valued relations were introduced by G. Birkhoff in 1948 as the best possible approximation to an inverse in the monoid of relations. We have generalized these concepts by introducing residuals of fuzzy relations. We show that residuals of fuzzy relations play a crucial role in computatuion of the greatest solutions to some systems of fuzzy relation inequalities and equations with significant applications in various fields, e.g., in automata theory, network analysis, etc. In general there is no residuation for matrices over a semiring, but we show that for matrices over an additively idempotent semiring there is a kind of relative residuation which allows to define and study Boolean residuals of matrices.

New convergences and topologies on (partial) function spaces

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One of the central questions of Analysis:was: what precisely must be added to pointwise convergence of a sequence of continuous functions to preserve the continuity of the limit? In 1841 Weierstrass discovered that uniform convergence yields continuity of the limit function. In 1983 Arzelà found a necessary and sufficient condition under which the pointwise limit of a sequence of real valued continuous on a compact interval is continuous. In 2009 Caserta, Di Maio, Hola survey a wide range of deep generalizations. In 2010 Caserta , Di Maio, Kocinac focus on recent directions in this area, e.g.the statistical approach . New notion of convergences for nets of partial functions were tackled by Beer, Caserta, Di Maio, Lucchetti in 2013.

Matrix pencils completion problems

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I will present recent advances on the general matrix pencils completion problem. This problem consists of describing the possible Kronecker invariants of a matrix pencil whose arbitrary subpencil is prescribed. Apart from general fundamental interest, this problem and its particular cases are of high importance in applications in control theory.In particular, I will present novel combinatorial techniques that we introduced, and that were crucial for resolving important particular cases of the general problem.

Random constructions imply symmetry

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We will argue for the claim of the title in the areas of algebra, theoretical computer science, and theoretical physics. In algebra, we will consider the random graph and Ulm's theorem for countable abelian p-groups. For theoretical computer science, we will give a probabilistic construction of Scott domains and show that with probability 1 our construction produces a universal homogeneous domain. Finally, we consider causal sets which have been used as basic models for discrete space-time in quantum gravity.

On methods of discrete search for singularities of mappings of metric spaces

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Given a finite collection of (one-valued or multivalued) mappings between metric spaces, the problem of search and approximation of some their singularities is considered. By singularities we mean here some special subsets such as common fixed point set, coincidence set, of common preimages of a given closed subspace, set of common roots and some others. We present some new results in this area based on the use of functionals strictly subjected to convergent series, which were recently introduced by the author.

Extreme solutions to inequalities and equations defined by residuated functions

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Residuated functions were introduced in the theory of ordered sets as a counterpart tocontinuous functions in mathematical analysis, as functions having the property that the inverseimage of a principal down-set is a principal down-set.We study inequalities and equations defined by residuated and residual (dually residuated)functions and we show that the problems of computing the extreme solutions to these equationsand inequalities boil down to the problems of computing the greatest post-fixed points or theleast pre-fixed points, using the well known Knaster-Tarski fixed point theorem. Our approachis very general and it includes, as special cases, many relation inequalities and equations alreadyknown in the literature, which will be considered here, too.

On graded Ω-groups

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We introduce the notion of a graded Ω-group, but graded in the sense of M. Krasner, that is, we do not assume neither the commutativity, nor the associativity, nor the existence of the neutral element in the set of grades. We prove that graded Ω-groups in Krasner's sense are determined up to isomorphism by their homogeneous parts, which, with respect to induced operations, represent general structures of their own called Ω-*homogroupoids*, thus narrowing down the theory of graded Ω-groups to the theory of Ω-homogroupoids. As an application, we discuss the theory of prime radicals for Ω-homogroupoids thus extending results of A. V. Mikhalev, I. N. Balaba and S. A. Pikhtilkov in a natural way.

On some mean value results for the zeta-function and a divisor problem

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Let $\Delta(x)$ denote the classical error term in the Dirichlet divisor problem, and let the modified error term in the divisor problem be $\Delta^*(x) = -\Delta(x) + 2\Delta(2x) - \frac{1}{2}\Delta(4x)$. It is shown that, for $T^{2/3+\epsilon} \le H \le T$,

$$\int_T^{T+H} \Delta^*\Big(\frac{t}{2\pi}\Big)|\zeta(1/2+it)|^2 dt \ll HT^{1/6}\log^{7/2} T.$$

We also obtain an asymptotic formula for

$$\int_0^T \Big(\Delta^*\Big(\frac{t}{2\pi}\Big)\Big)^2|\zeta(1/2+it)|^2 dt,$$

and for the integral with the cube of the Δ^*-function as well. The importance of the Δ^*-function comes from the fact that it is analogue of the error term in the mean square formula for $|\zeta(1/2+it)|^2$.

Effective Homology computation using symbolic software

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In this paper, the possibility of using the symbolic software for calculating homology is being considered. The accent is being put on applications and possibilities of using the Kenzo program.

Equivariant topology: action's types, their extensions

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The investigation of action's extensions started with the following compactification problem: whether any G-space is G-Tychonoff (has compact G-extension)? This problem is satisfactory solved. J.de Vries characterized G-Tychonoff spaces using uniform structures and a concept of bounded action. Sufficient condition (quasiboundedness of action) when the action can be extended over the completion of a phase space was introduced by M.Megrelishvili. Quasibounded actions generalize both bounded and uniformly equicontinuous ones. Moreover, quasiboundedness also guarantees the possibility of action's extension over the completion of the acting group in two-sided uniformity. The existence of uniformities on a G-space with respect to which the action is quasibounded characterizers the case when the G-space is G-Tychonoff.Boundedness, uniform equicontinuity and quasiboundedness of actions are characterized as action's uniform continuity on the (piecewise) semi-uniform product. From this point of view the origin of different examples of action's extensions are explained.In the study of semi-lattices of compact G-extensions results and examples when a semi-lattice has a minimal or the smallest element are presented. Compact G-extensions of h-homogeneous spaces, in particular of rationals, are discussed.

An Integer Sequence Derived by Polygon Triangulation

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In this paper we investigate the correlation between the convex polygon triangulations, the corresponding arithmetic expressions and the integers derived by a suitable mapping of these expressions. Further, some underlying results are proven and forbidden integer values are counted. Finally, another way to express Catalan number is derived.

Some applications of nonstandard analysis to functional equations

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In this talk we will present and discuss some nonstandard methods applied for solving certain functional equations. In particular, we will prove that all measurable solutions of the considered equations are continuous. Some applications will be given.

Joint discrete universality of the Riemann zeta-function and Hurwitz zeta-function

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In the report, we will present a joint discrete universality theorem for the Riemann zeta-function $\zeta(s)$ and Hurwitz zeta-function $\zeta(s,\alpha)$, $s=\sigma+it$, $0<\alpha\leq 1$. Let $D=\left\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\right\}$, $\mathcal{K}$ be the class of compact subsets of D with connected complements, $H_0(K)$, $K\in\mathcal{K}$, be the class of non-vanishing continuous functions on K which are analytic in the interior of K, and let $H(K)$, $K\in\mathcal{K}$, be the class of continuous functions on K which are analytic in the interior of K. Define the set $L(\mathcal{P},\alpha,h)=\left\{(\log p:p\in\mathcal{P}),(\log(m+\alpha):m\in\mathbb{N}_0),\frac{2\pi}{h}\right\}$, where $\mathcal{P}$ is the set of all prime numbers, and $h>0$ is a fixed number.**Theorem.** *Suppose that the set $L(\mathcal{P},\alpha,h)$ is linearly independent over the field of rational numbers, $K,K_1\in\mathcal{K}$, $f(s)\in H_0(K)$ and $f_1(s)\in H(K)$. Then, for every $\varepsilon>0$,*

$$\liminf_{N\to\infty}\frac{1}{N+1}\#\left\{0\leq k\leq N:\sup_{s\in K}|\zeta(s+ikh)-f(s)|<\varepsilon,\right.$$
$$\left.\sup_{s\in K_1}|\zeta(s+ikh,\alpha)-f_1(s)|<\varepsilon\right\}>0.$$

The theorem is a discrete version of the Mishou theorem proved for continuous shifts $(\zeta(s+i\tau),\zeta(s+i\tau,\alpha))$ with transcendental α.

On arithmetic operations on the set of pythagorean triples

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The problem of Pythagorean triples (or integer-sided right-angled triangles, so called Pythagorean triangles) has been attracting mathematicians for centuries. Although the first general solution to the problem was known already in Ancient Greece, mathematicians continued looking for new parametrisations of the problem. In this vein, one of the authors had previously described a new natural parametrisation of Pythagorean triples, based on the elements of the corresponding right-angled triangle. In the present paper we shall describe an attempt to introduce basic arithmetic operations on the set of all Pythagorean triples or on some of its characteristic subsets.

A note on multivariate polynomial division and Gröbner bases

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In this presentation, a new view of the multivariate division algorithm will be given and applied to the notion of Gröbner basis in the multivariate polynomial ring. This will lead to a new equivalent characterization of Gröbner basis.

On influence of probabilistic number theory to probabilistic combinatorics

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The central limit theorem for the number of different prime factors of a natural number (Erdös-Kac, 1939) and that for the number of cycles of a random permutation in the symmetric group (Goncharov, 1942) were established almost at the same time. Nowadays, being a bit ahead probabilistic number theory supplies ideas to carry out parallel theories on permutations, all mappings of a finite set into itself or general classes of decomposable structures like assemblies, weighted multisets *et cetera* (see [1]). Going along this path, we explore various statistics of random permutations σ taken at random from the symmetric group §. Some of the recent results will be discussed in the talk. For illustration, we present here an analog of the weighted Turán-Kubilius inequality.

Let $k_j(\sigma) \geq 0$, $1 \leq j \leq n$, be the number of cycles in the canonical decomposition of σ into a product, $w(\sigma) = k_1(\sigma)+\cdots+k_n(\sigma)$ be the number of all cycles, $\Theta^{(n)} := \theta(\theta+1)\cdots(\theta+n-1)$ where $\theta > 0$ is a constant. The Ewens probability measure in § is defined by $\nu_n(\{\sigma\}) = (\Theta^{(n)})^{-1}\theta^{w(\sigma)}$. By definition, a real additive function has an expression $h(\sigma) = h_1\big(k_1(\sigma)\big) + \cdots + h_n\big(k_n(\sigma)\big)$, where $h_j(s) \in R$ and $h_j(0) \equiv 0$. For its variance with respect to ν_n, we have

$$\mathbf{V}_n h(\sigma) \leq C(\theta) \sum_{jk\leq n} \Big(\frac{\theta}{j}\Big)^k \frac{h_j(k)^2}{k!} \Big(1 - \frac{jk}{n+1}\Big)^{\theta-1}.$$

The ideas of Biró and Szamuely [2] lead to further inequalities with multiplicative weights.

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Hyperspaces of 0-dimensional spaces revisited

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All spaces under consideration are supposed to be compact and Hausdorff. For a space X, its hyperspace $exp(X)$ is the set of all non-empty closed subsets of X taken with the Vietoris topology. Although in 1920s, the famous conjuncture: for a non-degenerate Peano continuum X, $exp(X) \approx Q$ (Q being the Hilbert cube) was launched, decades had passed before the first concrete examples of hyperspaces were discovered. The class of all compact 0-dimensional metric spaces $\mathbf{Z}$ is another natural framework for trying to fix the topological types of hyperspaces. Let $C_{-1} = \emptyset, C_0 = 1, C_1 = C$ (Cantor discontinuum). A sequence of very regular topologically distinct spaces in $\mathbf{Z} : C_0, C_1, C_2, \ldots, C_n, \ldots$ is constructed inductively taking C_n to be the space C together with small enough copies of $C_{n-3} \oplus C_{n-2}$ interpolated in each of its removed intervals. In 1964, A Pelczynski proved that for each X in $\mathbf{Z}$, having the set of its isolated points everywhere dense, $exp(X) \approx C_2$. In 1972, we successed to prove that for each X in $\mathbf{Z}$ (excludingthe trivial cases of spaces having finite number of isolated points), $exp(X)$ is one of the following spaces: $C_1, C_2, C_1 \oplus C_2, C_3, C_4, C_5, C_7$. In the same 1972, R. Schori, D.Curtis and J. West proved the most significant result on hyperspaces, confirming the above mentioned conjecture. In 2005, S. Oka (Topology and its Applications, 149, p. 227–237) reproves our result from 1972, without referring to our paper. That motivates me for this gripping narrative on hyperspaces.

Semiregular subgroups of transitive permutation groups

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In this talk I will discuss the semiregularity problem which in its original form asks if every vertex-transitive digraph has a nonidentity semiregular automorphism, that is an automorphism with all orbits of the same size. I will give a short overview of the current status of this problem (adn some of its generalizations) and discuss possible future directions.

On Kurepa's determinants arising from Kurepa's left factorial hypothesis

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Kurepa's (left factorial) hypothesis asserts that for each integer $n \geq 2$ the greatest common divisor of $!n := \sum_{k=0}^{n-1} k!$ and $n!$ is 2. It is known that Kurepa's hypothesis is equivalent to

$$\sum_{k=0}^{p-1} \frac{(-1)^k}{k!} \not\equiv 0 \pmod{p} \quad \text{for each prime} \quad p \geq 3.$$

Motivated by the above reformulation of Kurepa's hypothesis, and using a Linear Algebra approach, for every integer $n \geq 7$ we define the so called Kurepa's determinant K_n of order $n-4$. We prove that Kurepa's hypothesis is equivalent with the assertion that $K_p \not\equiv 0 (\mathrm{mod}\, p)$ for all primes $p \geq 7$. Furthermore, we establish some interesting divisibility properties of Kurepa's determinants K_n with composite integers $n \geq 8$. Related computational results are also presented.

Definable subsets of finite structures and applications

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Let $\mathbf{A}$ be a model of a first-order language L and $X \subseteq A$. We say that X is definable in $\mathbf{A}$ if $X = \{a \in A | \mathbf{A} \models \varphi(a)\}$ where $\varphi(x)$ is a formula of L. Also, X is absolutely invariant in $\mathbf{A}$ if for all $f \in \mathrm{Aut}(\mathbf{A})$, $f(X) \subseteq X$. Using Svenonius definability theorem [], we describe various definable subsets of a finite model $\mathbf{A}$. For example we proved:

Theorem. *Let $\mathbf{A}$ be a finite model of L and $X \subseteq A$. Then X is absolutely invariant in $\mathbf{A}$ if and only if X is definable in $\mathbf{A}$.*

Corollary. *Let $\mathbf{A}$ be a finite model of finite L. If $a \in A$ is fixed by all automorphisms of $\mathbf{A}$ then a is definable in $\mathbf{A}$ by a formula $\varphi(x)$ of L. $\mathrm{Aut}(\mathbf{A}) = \{i_A\}$ if and only if every element of A is definable in $\mathbf{A}$.*

Using definability theory of finite structures, we developed a parallel program for computing related structures to finite models. Examples include finite labeled and unlabeled models of a first order theory, their numbers and $\mathrm{Aut}(\mathbf{A})$ of a finite model $\mathbf{A}$.

Ž. Mijajlović, A. Pejović, *Computing finite models using free Boolean generators*, arXiv:1310.6978, 2013.

L. Svenonius *A theorem on permutations in models*, Theoria 25(3), 173-178, 1959.

Subsets of the reals that are countable dense homogeneous

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A space X is countable dense homogeneous (CDH) if for all countable dense subsets D and E of X there is a homeomorphism $f : X \to X$ such that $f(D) = E$. In this talk we are interested in CDH-subsets of the reals. The spaces we are interested in have no isolated points. No such countable space is CDH. It is known that every Borel CDH-subset of the reals is Polish. It is also known that there is a CDH-subset of the reals of cardinality $\aleph_1$. We show that sufficiently nice λ-sets are CDH. We will also show that there is a Baire CDH-subset of the reals that is not Polish. This is joint work with Michael Hrušák and Rodrigo Hernandez-Gutiérrez.

Kings in hypertournaments

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Hypertournaments are one of many possible generalizations of tournaments. In the talk the existence and the distribution of special vertices of hypertournaments, called kings, will be discussed.

Simplicial complexes associated to commutative rings

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The topic of graphs (1-dimensional complexes) associated to commutative rings is well known. Here we review some interesting results and present some new ones, including higher dimensional complexes associated to commutative rings.

New results on the difference of consecutive primes

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We will give a short overview of the recent breakthrough results of Y. Zhang, J. Maynard and T. Tao proving that there are infinitely many bounded gaps between consecutive primes. The results of Maynard and Tao even imply that there are arbitrarily long finite blocks of consedcutive primes in bounded intervals infinitely often. We will mention several other results which refer to so called Polignac numbers which have the property that they appear infinitely often as difference between consecutive primes. We also mention answers on several 60-70 years old conjectures of Erdös referring for the difference of consecutive primes. Finally we mention that the celebrated theorems of Green-Tao and Zhang-Maynard-Tao can be generalized in the way that there exists a bounded difference d with the property that there are arbitrarily long finite arithmetic progressions of primes such that p+d is the prime following p for every element of the progression.

A description of the cohomology of real flag manifolds

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By Borel's description, the mod 2 cohomology of a real flag manifold is isomorphic to a polynomial algebra modulo a certain ideal I. In this talk we present a new generating set G for I. Furthermore, we pose a conjecture on the additive basis of this cohomology (this conjecture is true for many special types of flag manifolds, for example Grassmann manifolds) and prove that G is a Gröbner basis for I if and only if this conjecture is true.

Koszul rings

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Some examples of Koszul rings will be presented.

On The Some Set Theoretical Properties of Logical Consequence

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The logical consequence undisputedly is the central concept of logic. The main purpose of logic is to tell us what follows logically from what. Logical consequence is a relation between a given set of formulas and the formulas that logically follow. In this work, we inspect set theoritical properties of the logical consequence related to denumerable sets and independet sets formulas of classical logic. We show that that intersections of independent sets are independent but union of two independent sets is not in general independent.

Intrinsic shape - the proximate approach

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In the paper will be shown that proximate fundamental group introduced in [2] is an invariant of pointed intrinsic shape, and will be introducedthe higher proximate groups.

References:
[1] N. Shekutkovski, Intrinsic definition of strong shape for compact metric spaces, Topology Proceedings 39, 2012, 27-392.
[2] Proximate fundamental group, N. Shekutkovski, A. Velkoska, Proceedings of the FMNS 2013, Volume 1

One Invariant of Intrinsic Shape

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Based on the intrinsic definition of shape by functions continuous up to a covering and corresponding homotopy we will provide new definition of pointed proximate nets and pointed homotopy up to a covering into paracompact spaces. These definitions are necessary for introducing Pointed Homotopic Up to a Covering Category. In a paper "Pointed Intrinsic shape", N. Shekutkovski, A. Velkoska, Proceedings of the FMNS2013, Volume 1, June 2013 we defined proximate fundamental group, an invariant of pointed shape of a space. In this paper we will show that that group is an invariant of Intrinsic Shape

Roots of trinomials of bounded height

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Lind and Boyd conjectured that the smallest Perron number of degree n is a root of a trinomial with coefficients 1, -1. We investigate some properties of these trinomials, such as number of roots which are greater than one in modulus and Mahler measure. We determine the limit of the rate $\frac{\nu}{n}$ between the number ν of roots of the trinomial $x^n - x^m - 1$, $0 < m < n$, which are greater than 1 in modulus, and degree n. The product of these ν roots has also a limit when $n \to \infty$. The explicit expression of the limit by an integral is presented. The computing of the rate and the product for $n = 100, 150$ as well as of its limits is presented.

On the limit distributions for some sets of additive arithmetic functions

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The limit behaviour of distributions $(1/[x])\sum_{\substack{n\le x\\ f_x(n)-\alpha(x)<u}} 1$ was considered in the probabilistic number theory very often.There were considered various classes of additive functions $f_x(n) = \sum_{p^r||n} f_x(p^r)$ with different centeringfunctions $\alpha(x)$. But the class of examined additivefunctions f_x was of a special expression:

$$f_x(n) = h(n)/\beta(x), \tag{1}$$

where f is some additive function and $\beta(x)$ is someunboundedly increasing function. In the books [1] and [2], and works cited there, one canfind almost all classical results and their historical context.An object of our talk is strongly additive functions taking thevalues 0 or 1 on the set of primes and maybe depending on x(therefore we call usually f_x by a set of additive functions), and in general case it isimpossible to express f_x by relation (1). We willdiscuss about the weak convergence of distributions

$$(1/[x])\sum_{\substack{n\le x\\ f_x(n)<u}} 1$$

for the set of strongly additive functions f_x as $x \to \infty$and about the class of possible limit laws.

References:
[1] P.D.T.A. Elliott, *Probabilistic Number Theory, I*,Springer-Verlag, New York, 1979, *Probabilistic NumberTheory, II*, Springer-Verlag, New York, 1980.
[2] J. Kubilius, *Probabilistic Methods in the Theory of Numbers*, Providence, Amer. Math. Soc. Transl. of Math. Monographs, **11**, 1964.

MathChem package for topological indices and its research uses

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We present MathChem, an open-source and cross-platform Python package, aimed at supporting research in mathematical chemistry. MathChem enables researchers to load batches of molecules or molecular graphs, calculate topological indices, perform statistical analyses and visualize the results. As a Python package, MathChem is easily integrable with Sage and other Python libraries such as NumPy and SciPy, which offer numerous further possibilities for analysis of calculated data. We present an example of such use by determining networks of correlations between various topological indices, which lead to an unexpected conjecture on relation between the graph energy and the atom-bond-connectivity index.

Combinatorial Hopf algebra of matroids

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Matroids are combinatorial structures that capture the notion of independence. The combinatorial Hopf algebra of matroids have been recently introduced by Billera, Jia and Reiner. The assigned quasisymmetric function is a combinatorial invariant of matroids. We study this invariant under matroid operations.

Coherent omission of intervals: A combinatorial method for constructing special sets of reals and associated topological spaces

Boaz Tsaban
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I will describe a combinatorial method for constructing subsets of the real line with strong (selective) topological properties.This method synthesizes and extendsideas from the Galvin–Miller classical construction ofa γ-set and the Bartoszyński–Shelah construction of aHurewicz non-σ-compact set of reals.This method lead to solutions of some notoriousproblems concerning the topology of the real line. Some of its major results are:

1. A construction of a nontrivial γ-set of realsfrom a weaker hypothesis than all those hitherto used(some axiom is necessary).
2. There is a non-σ-compact, *productively* Hurewicz set of reals.
3. There is (in ZFC) a set of reals, of the uncountable cardinality $\mathfrak{b}$, such that for each sequence of open covers, one can choose for each n two elements U_n and V_n fromthe n-th cover, such that the sequence $U_1 \cup V_1, U_2 \cup V_2, \ldots$ is a point-cofinite cover of our set.
(*Two* cannot be provably reduced to *one* here.)

This method shows that very natural constructions produce such sets.Credits: Item (1) is joint work with Tal Orenshtein. Item (2) is joint workwith Lyubomyr Zdomskyy. The nice name *coherent omission of intervals*for the method I propose was coined by Zdomskyy.

Braids and Algebraic Topology

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We start with the definition of braid group as the fundamental group of configuration space and as the mapping class group of a disc with punctures. Braids admit generalizations in various directions. The are also special types of braids defined among all braids by specific properties. One of natural generalizations of braids is the monoid of partial braids. We interprete it as a monoid of certain isotopy classes of homeomorphisms of punctured disc. Brunnian braid is a braid that becomes trivial after removing any one of its strands. We consider Brunnian braids on surfaces. In the cases of sphere and projective plane Brunnian braids are connected with homotopy groups of 2-dimensional sphere. Cohen braids are related to Brunnian braids and we study their properties also.

Compactifications and remainders of Tychonoff spaces

Alexander Vladimirovich Arhangelskii
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The search for "nice" compactifications of topological spaces is a natural direction of research in Topology. It is, probably, even more natural to look for compactifications with remainders satisfying certain special requirements. Some recent results of this kind will be described, and certain attractive open questions will be formulated.

References:
[1] Arhangel'skii A.V., Remainders of metrizable and close to metrizable spaces, Fundamenta Mathematicae 220 (2013), 71-81.
[2] Arhangel'skii A.V., A generalization of Cech-complete spaces and Lindelof Σ-spaces Comment. Math. Universatis Carolinae 54:2 (2013), 121-139.

On Tong's method and its applications

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In 1956 Tong studied the mean square of the error term in the general Piltz divisor problem and established a true asymptotic formula in the three-dimensional case. In this talk I will describe Tong's method for a class of general arithmetic functions. Some applications will be given.

Field 2:

Numerical Mathematics, Applied Mathematics

q-analogues of multiparameter non-central Stirling numbers

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In this paper we derive q-analogues of the multiparameter non-central Stirling numbers of the first and second kind. Moreover, recurrence relations, explicit formulas and a connection between these numbers and generalized q-harmonic numbers are obtained. Furthermore, some important special cases and new combinatorial identities are given. Finally, algorithms of these numbers and matrix representation using maple are derived.

A Finite Difference Scheme for a Fractional Super-diffusion Equation with Concentrated Capacity

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In this paper we consider finite-difference scheme for a fractional super-diffusion equation with the coefficient at the time derivative containing Dirac delta distribution. The stability and convergence of the scheme are proven using the discrete energy method. Estimate for the rate of the convergence compatible with the smoothness of the solution is obtained. A numerical exampledemonstrates the theoretical results.

New Hybrid Conjugate Gradient Method for Unconstrained Optimization

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A new hybrid conjugate gradient algorithm is considered. We require a specific determination of parameters. The parameter β_k is computed as a convex combination of β_k^{LS} (Liu and Storey) and β_k^{CD} (Conjugate Descent). We compute the parameter θ_k in such a way that the conjugacy condition is satisfied. The standard Wolfe line search conditions are used. Numerical comparisons show that the present hybrid conjugate gradient algorithm often behaves better than some previous algorithms.

Petrovic's Elements of mathematical phenomenology AND Phenomenological Mappings in Science

Katica R.(Stevanovic) Hedrih

Mathematical Institute SANU Belgrade, Department for Mechanics and Faculty of Mechanical Engineering, University of Niš, Serbia, khedrih@sbb.rs

Petrovic's Elements of Mathematical Phenomenology, from 1911, is published on 789 pages and Phenomenological Mappings, issue from 1933 on 33 pages, contain fundamental and basic ides, which from time to time appear in current scientific publications. These cited Petrovic's publications contain general theory of Elements of Mathematical Phenomenology and Phenomenological Mappings, founded in 1911, but both books was published in Serbian and no visible from other word languages. Petrovic Mihaillo (1868-1943) [3] was Serbian scientist, and previously French Sorbonne student of famous scientists as Poincaré, Appall, Hermite, Picard, Painlevé, Bousinesq and other.

An approach to integration and reductions of models of dynamics in different area of sciences on the basis of qualitative, structural and mathematical analogies is presented. Some examples of qualitative /mathematical analogies on the basis of generalized Lissajous curvesore identify and presented. Qualitative /mathematical analogies between eigne main fractional order modes in dynamics of different physical fractional order systems are presented.

About a nonlocal BVP for Poisson's equation

Sandra Hodzic

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In this paper we consider a nonlocal boundary-value problem for Poisson's equation on the unit square. A priori estimate for its weak solution in appropriate Sobolev space is proved. A finite difference scheme approximating this problem is proposed. An error estimate in a discrete W_2^1 Sobolev norm is obtained.

On graded Ω-groups

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We introduce the notion of a graded Ω-group, but graded in the sense of M. Krasner, that is, we do not assume neither the commutativity, nor the associativity, nor the existence of the neutral element in the set of grades. We prove that graded Ω-groups in Krasner's sense are determined up to isomorphism by their homogeneous parts, which, with respect to induced operations, represent general structures of their own called Ω-*homogroupoids*, thus narrowing down the theory of graded Ω-groups to the theory of Ω-homogroupoids. As an application, we discuss the theory of prime radicals for Ω-homogroupoids thus extending results of A. V. Mikhalev, I. N. Balaba and S. A. Pikhtilkov in a natural way.

A new proof of the necessary conditions of the first order for the general problem of the calculus of variations

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The biggest difficulties in the proofs of necessary conditions of the first order for problems in the calculus of variations are caused by a differential equation appearing in the constraints. The right hand side of this equation depends on control, so its maximal solution depends on it too, not only on the initial conditions. We prove that the domain of the maximal solution is an open set, that it has continuous partial derivatives in all variables, and we find its partial derivatives. Using maximal solution of the differential equation, we transform the general problem of the calculus of variations into a smooth problem of mathematical programming. Applying the corresponding necessary conditions of the first order, we obtain the necessary conditions of the first order for the general problem of the calculus of variations

Some Boundary Value Problems for Fractional in Time Diffusion-Wave Equation and their Numerical Solution

Bosko Jovanovic
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The use of fractional partial differential equations in mathematical models has become increasingly popular in the last decade. Unlike the classical derivatives, fractional order derivatives are non-local operators. This property can be interpreted as a type of memory effect which is characteristic for different materials and processes. This explains one of the most significant uses of fractional PDE in applications. The same feature, however, makes difficult the design of fast and accurate numerical methods for such type of equations.In this paper we present some examples of fractional in time diffusion-wave equation and highlight the main theoretical and numerical problems appearing. In particular, we introduce some interface and transmission problems related with such type of equations.

A new computational tool for option replication

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In this work we present a new computational tool for option replication. Specifically, we propose new methods for computing the replicated exercise prices of a given portfolio under a matrix-based framework. Our analysis involves the theory of positive bases in Riesz spaces.

Spectral gradient method for stochastic optimization

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We consider constrained optimization problems with the objective function in a form of mathematical expectation and convex, compact feasible set. At every iteration, the proposed method uses a sample average in order to approximate the objective function. Sample size is updated at every iteration and it does not have to be monotonically increasing. Line search along the direction of a spectral projected gradient is employed. Feasibility is maintained throughout the whole optimization process. Almost sure convergence is analyzed.

Inexact Restoration approach for minimization with inexact evaluation of the objective function

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A new method is introduced for minimizing a function that can be computed only inexactly, with different levels of accuracy. The challenge is to evaluate the (potentially very expensive) objective function with low accuracy as far as this does not interfere with the goal of getting high accuracy minimization at the end. For achieving this goal the problem is reformulated in terms of constrained optimization and handled with an Inexact Restoration technique. Convergence is proved and numerical experiments motivated by the Schrodinger Equation in Electronic Structure Calculations are presented, which indicate that the new method overcomes current approaches for solving large-scale problems.

One method for proving a class of trigonometric inequalities

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In this paper is considered one method for proving of a class of trigonometric inequalities. Analysis of application of method on some results from theory of analytic inequalities was performed, as well as connection with known automatic theorem proverssuch as MetiTarski (L.C.Paulson).

Plane curves with foci and directrices in the facility location problems

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In this paper we present a generalization of weighted k-ellipses. Let us give an n-foci (points) and an m-directrices (lines)in a Euclidean plane for $n, m \in \mathbb{N}_0$.In this plane for the point W we consider the following equation

$$\sum_{i=1}^{n} \alpha_i R_i + \sum_{j=1}^{m} \beta_j r_j = S$$

where R_i is Euclidean distance from the point W to the i^{th} focus, r_j is Euclidean distance from the point W to the j^{th} directrix and $\alpha_i, \beta_j, S \in \mathbb{R}$. We present some statements about existence of the real solutions of previous equation. Some well-known examples of these types of plane curves with foci and directricesare conic sections,weighted k-ellipses, weighted multidiretrices polygonals and Erdös-Mordell curve. An algorithm for extraction of locus of W-points as a part of appropriate algebraic curves of higher order is presented. For some applications of W-curves in the facility location problems,web-based program was developed.

A Motzkin-type theorem of the alternative in L_p spaces

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A theorem of the alternative (or a transposition theorem) asserts that two systems are related in such a way that exactly one of them is consistent. These theorems play a important role in deriving optimality conditions for wide classes of extremal problems. We will present some generalizations of well known Motzkin's theorem of the alternative, i.e., we focus on Motzkin's theorem that represents a continuous-time analogue of the original theorem.

A continuous-time generalization of Motzkin's theorem of the alternative

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A theorem of the alternative (or a transposition theorem) assertsthat two systems are related in such a way that exactly one of themis consistent. These theorems play a important role in derivingoptimality conditions for wide classes of extremal problems. We willpresent some generalizations of well known Motzkin's theorem of thealternative, i.e., we focus on Motzkin's theorem that represents acontinuous-time analogue of the original theorem.

Modelling uncertainties in structural design and assessment

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Uncertainties associated with structural design and assessment stem from variability in material properties, geometry, simplifications in load models, diverse structural behaviour models, etc. With increasing requirements for risk evaluation for infrastructure ever more sophisticated methods are needed to deal with relevant uncertainties and likelihood of critical events. Practical formulation for design and assessment of built infrastructure will be identified both for ultimate and serviceability limit states. Using an example of steel portal frame specific techniques used for uncertainty modelling for built infrastructure will be reviewed and future trends identified. Modelling for twin problem of ageing infrastructure and changing environmental conditions (climate change in particular) will be addressed.

Fast summation methods based on quadratures of Gaussian type

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An account on summation/integration methods for computation of slowly convergent series and finite sums are given. Methods are based on Gauss quadrature formulas with respect to some nonclassical weight functions over real (half) line. A recent progress in symbolic computation and variable-precision arithmetic are used in the construction of such quadrature rules. Some interesting special cases are presented.

Numerical approximation of 2D elliptic transmission problem in disjoint domains

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An elliptic transmission problem in disjoint domains is investigated. The existence and uniqueness of its weak solution is proved. A finite difference scheme approximating this problem is proposed and analyzed. An estimate of the convergence rate, compatible with the smoothness of the input data (up to a logarithmic factor of the mesh size) is obtained.

Image restoration methods based on least squares solutions

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In this work we examine the problem of image restoration methods. The methods that will be presented generalize image restoration algorithms that are based on the Moore-Penrose solution of certain matrix equations that define the linear motion blur. Our approach is based on the usage of least squares solutions of these matrix equations, where an arbitrary matrix of appropriate dimensions is included besides the Moore-Penrose inverse. The arbitrary matrix is replaced by the results given by some of known image restoration methods, such as moment based methods (the Haar basis and Fourier basis). In addition, the method is a useful tool for improving results obtained by other image restoration methods. The methods have been tested by reconstructing an image after the removal of blur caused by uniform linear motion or by separable motion blur, filtering the noise that is corrupted with the image pixels. The quality of the restoration is observable by a human eye. Benefits of using the presented methods are illustrated by the values of the improvement in signal-to-noise ratio (ISNR) and in the values of peak signal-to-noise ratio (PSNR).

Outer Inverses in constrained quadratic optimization

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In this work we examine the problem of minimization of a positive semidefinite quadratic form under linear constraints. We introduce the use of various kinds of outer generalized inverses, define such outer inverses depending on the constraint set and present their properties. In addition, the minimizing vector of this problem has an additional property: it should belong to the set perpendicular to the kernel of the quadratic form examined. Finally, the T-restricted outer inverse of a matrix is introduced, covering all the cases presented in several published papers.

Hankel transform computation of different integer sequences

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For a given sequence a_n, the sequence of Hankel determinants $h_n = \det[a_{i+j}]_{0\leq i,j\leq n}$ is known as Hankel transform. We consider the Hankel transform evaluation of a sequences based on the Motzkin and Catalan numbers. Computation is done using the method based on orthogonal polynomials (continued fractions) and transformations of the weight function. It is also investigated the connection between Hankel and aerating transformations.

Family of simultaneous methods with corrections for approximating zeros of analytic functions

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A family of iterative methods for the simultaneous determination of complex zeros of a class of analytic functions, is proposed. Considered analytic functions have only simple zeros inside a simple smooth closed contour in the complex plane. We show that the convergence of the basic family of the fourth order can be increased to five and six using Newton's and Halley's corrections, respectively. The improved convergence is achieved with negligible number of additional calculations, which significantly increases the computational efficiency of the accelerated methods. Numerical example demonstrate a good convergence properties, fitting very well theoretical results.

Analogue of Gauss-Lucas theorem

Blagovest Sendov
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Let $S(\phi) = \{z : |\arg(z)| \geq \phi\}$ be a sector on the complex plane $\mathcal{C}$. If $\phi \geq \pi/2$, then $S(\phi)$ is a convex set and, according to the Gauss-Lucas theorem, if a polynomial $p(z)$ has all its zeros on $S(\phi)$, then the same is true for the zeros of all its derivatives. The main theorem in this lecture, called **Sector theorem**, asserts that if the polynomial $p(z)$ is with real and non negative coefficients, then the same is true also for $\phi < \pi/2$, when the sector is not a convex set on the complex plane.

The Sector theorem is applied to prove stronger Rolle's theorem for complex polynomials.

Multiple orhogonality in the space of trigonometric polynomials if semi–integer degree

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In this paper we consider multiple orthogonal trigonometric polynomials of semi–integer degree, which are necessary for constructing of an optimal set of quadrature rules with an odd number of nodesfor trigonometric polynomials in Borges' sense [Numer. Math. **67** (1994), 271–288]. We prove that such multiple orthogonal trigonometric polynomials satisfy certain recurrence relation and present numerical method for their construction, as well as for construction of mentioned optimal set of quadrature rules. Theoretical results are illustrated by some numerical examples.

Computation of Generalized Matrix Inverses via Full-rank LDL* Decomposition

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We investigate a method of the full-rank LDL* factorization, where matrices L and D are abbreviated by zero rows and columns. Therefore, we provide explicit formulaes for the coefficients appearing in rational matrices L and D. An algorithm for the computation of $A^{(2)}_{T,S}$ inverses of a given matrix A, based on the full–rank LDL^* decomposition of an appropriate matrix M, is derived. This method is extended to the set of one–variable polynomial matrices, aiming to efficiently compute $A^{(2)}_{T,S}$ inverses of polynomial Hermitian matrices. We explain the implementation details in programming language MATHEMATICA, illustrate our algorithms via examples and compare them to other well-known methods.

Boolean Dierential Equations - Solution and Applications

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The *Boolean Differential Calculus* (BDC) significantly extends the Boolean Algebra because not only Boolean values of the set $\mathbb{B} = \{0, 1\}$, but also changes of Boolean values or Boolean functions can be described. A *Boolean Differential Equation* (BDE) is a Boolean equation that includes derivative operations of the Boolean Differential Calculus. This paper aims at the classification of BDEs, the characterization of the respective solution, algorithms to calculate the solution of a BDE, and selected applications. In order to reach this aim, we give a short introduction into the BDC, emphasize the general difference between the solutions of a Boolean equation and a BDE, explain the core algorithm to solve a BDE that is restricted to all vectorial derivatives of $f(\mathbf{x})$. We explain formulas for transformation of other derivative operations to vectorial derivatives in order to solve more general BDEs. Selected from the wide field of applications, we show BDEs which solve important tasks of circuit design and cryptography. The basic operations of XBOOLE are sufficient to solve BDEs. We demonstrate how a XBOOLE problem program (PRP) of the freely available XBOOLE-Monitor quickly solves some BDEs.

Quadrature rules with an even number of multiple nodes and a maximal trigonometric degree of exactness

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This paper is devoted to the interpolatory quadrature rules with an even number of multiple nodes, which have the maximal trigonometric degree of exactness. For the constructing of such quadrature rules we introduce and consider the so-called s- and σ-orthogonal trigonometric polynomials. We present a numerical method for construction of mentioned quadrature rules and some numerical examples are also included.

Stohastic process representation for time-dependent timber-concrete composite beam deterioration

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We consider a new stochastic process model that will capture the true nature of deterioration of timber-concrete composite beams. In order to enable efficient management of structures in terms of required maintenance, repair and/or replacement, it is essential to be able to capture the uncertain nature of the deterioration process. As the increasing deflection of the timber-concrete composite beam over time is generally uncertain and non-decreasing, it can best be regarded as a continuous gamma process.

Field 3:

Real and Functional Analysis, Differential Equations, Complex Analysis, Probability and Statistics

H-distributions

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H-measures (also known as microlocal defect measures) of Tartar [1] and Gérard [2] obtained for weakly convergent sequences in L^2, and their generalization to L^p, called H-distributions [3], are widely used to determine weather a weakly convergent sequence converge strongly.We extend the concept of H-distributions to the Sobolev spaces and give necessary and sufficient condition so that the weak convergence in $W^{-k,p}$, $p \in (1, \underline{)}$, implies the strong one.

References :

[1] Tartar, L.H-measures, a new approach for studying homogenisation, oscillations and concentration effects in partial differential equations.Proc. Roy. Soc. Edinburgh Sect. A 115 (1990), no. 3-4, 193–230.
[2] Gérard, P.Microlocal defect measures.Comm. Partial Differential Equations 16 (1991), no. 11, 1761–1794.
[3] Antonić, N., Mitrović, D. H-distributions: an extension of H-measures to an $L^p - L^q$ setting. Abstr. Appl. Anal. 2011, Art. ID 901084, 12 pp.

Construction of p frames for weighted shift-invariant spaces

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We construct a sequence $\{\phi_i(\cdot - j) \mid j \in \{ZZ\}, i = 1, \dots, r\}$ which constitutes a p-frame for the weighted shift-invariant space

$$V_\mu^p(\Phi) = \Big\{ \sum_{i=1}^{r} \sum_{j \in \mathbb{Z}} c_i(j)\phi_i(\cdot - j) \,\Big|\, \{c_i(j)\}_{j\in\mathbb{Z}} \in \ell_\mu^p,\ i = 1, \dots, r \Big\},\ p \in [1, \infty],$$

and generates a closed shift-invariant subspace of $L_\mu^p(\mathbb{R})$. The first construction is obtained by choosing functions ϕ_i, $i = 1, \dots, r$, with compactly supported Fourier transforms $\widehat{\phi}_i$, $i = 1, \dots, r$. The second construction,with compactly supported $\phi_i, i = 1, ..., r$, gives the Riesz basis.

Approximation for periodic functions via generalized weighted Norlund–Euler statistical convergence

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Korovkin type approximation theorems are useful tools to check whether a given sequence positive linear operators on of all continuous functions on the real interval is an approximation process. That is, these theorems exhibit a variety of test functions, which assure that the approximation property holds on the whole space if it holds for them. Such a property was discovered by Korovkin in 1953 for the functions and in the space as well as for the functions, and in the space of all continuous-periodic functions on the real line. In this paper, we use the notion of weighted Norlund-Euler statistical convergence to prove the Korovkin approximation theorem for the functions, and in the space of all continuous-periodic functions on the real line and show that our result is stronger. We also study the rate of weighted Norlund-Euler statistical convergence.

Maximum Principles for Some Higher Order Ordinary Differential Equations

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In this paper we establish maximum principles which applies on solutions of some third and fourth order ordinary differential equations.

Multipliers in spaces of harmonic functions and embeding theorems

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We give characterizations of spaces of multipliers acting between harmonic functions spaces on the unit ball. The spaces involved are mixed norm spaces, including Tiebel - Lizorkin spaces, Hardy and Besov spaces. The results presented in many cases generalize known results in the case of analytic function spaces on the unit disc. All of these results are obtained in collaboration with Romi F. Shamoyan.

Covering mappings and coincidence points

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Coincidence points of two mappings acting in metric spaces are studied. The sufficient conditions for existence of coincidence points of covering and Lipschitz mappings will be presented. Furthermore, there will be introduced some results on properties of coincidence points set.

A mean value theorem for system of integrals and the Gauss-Hermite quadrature

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A mean value theorem for system of integrals claims that, given any set of continuous functions on $I \subseteq R$, and a finite measure μ on I, there exists an n-point quadrature rule which is exact for those functions. If we consider μ as a Gaussian measure on the Borel sigma-field of R, we can use the Hermite polynomials to determine easily the nodes and coefficients of this quadrature rule. The case when measure μ is introduced by a Gaussian stochastic process has been investigated.

Uniqueness Of Meromorphic Functions Sharing Two Sets With Finite Weight III

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Lahiri [Weighted sharing and uniqueness of meromorphic functions, Nagoya Math. J., 161(2001), pp.193-206.] introduced the denition of gradation of sharing known as weighted sharing of values and sets which measure how close a shared value is to being shared CM or to being shared IM.
With the notion of weighted sharing of sets this paper investigate the problem of uniqueness of meromorphic functions sharing two sets which improve and supplement some existing results of Lahiri [On a question of Hong Xun Yi, Arch. Math. (Brno), 38(2002), 119-128.],
Banerjee [Uniqueness Of Meromorphic Functions Sharing Two Sets With Finite Weight, Portugal. Math. J., 65(1), 2008, 81-93.] and
yi and Lin [Uniqueness of meromorphic functions and a question of Gross, Kyungpook Math. J, 46 (2006), 437-444.]. We exhibit two examples to show that the condition over decient values which plays a key factor in the paper is the best possible.

Asymptotic behavior of function mean values

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A function $f(x) > 0$, $x > 0$ defines Moskovitz f-mean value of a and b [1,2]

$$M_f(a,b) = \frac{af(b) + bf(a)}{f(a) + f(b)}.$$

so that $(a, f(a))$, $(M_f(a,b), 0))$ and $(b, -f(b))$ are colinear.

Divided differences of the power means for equal values of variables [3] have C^∞ extension

$$\frac{M_p(\mathbf{x}) - M_q(\mathbf{x})}{M_r(\mathbf{x}) - M_s(\mathbf{x})} \to \frac{p-q}{r-s}, \quad \mathbf{x} \to \mathbf{a}, \quad \mathbf{x} = (x_1, \ldots, x_n), \quad \mathbf{a} = (a, \ldots, a).$$

Eg. the more general quasi-arithmetic means $M(\varphi; \mathbf{x}) = \varphi^{-1}\left(\sum u_i \varphi(x_i)\right)$, $u_i \geq 0$, $\sum u_i = 1$, have extension

$$\frac{M(\varphi; \mathbf{x}) - A(\mathbf{u}; \mathbf{x})}{Q(\mathbf{u}; \mathbf{x}) - A(\mathbf{u}; \mathbf{x})} \to \frac{\varphi''(a)}{\varphi'(a)} \cdot 2a, \quad \mathbf{x} \to \mathbf{a},$$

where $Q = M_2$ and $A = M_1$ are the quadratic and arithmetic mean.

Theorem 1.

$$\frac{M_f(x_1, x_2) - A(x_1, x_2)}{Q(x_1, x_2) - A(x_1, x_2)} \to -\frac{f'(a)}{f(a)} \cdot 2a, \quad (x_1, x_2) \to (a, a).$$

The proof follows by Taylor's expansion. If $f(x)$ is 1, $\sqrt{x}$ and x, then f-means become $A(a,b)$, $G(a,b)$ and $H(a,b)$ respectively; $G = M_0$ and $H = M_{-1}$ are the geometric and harmonic means. For these three cases the above extensions are identical.

References:
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[3] M. Bjelica, *Asymptotic linearity of mean values*, Matematički Vesnik **51** 1999, Symposium on Mathematical Analysis and its Applications, 15–19.

Principal Component Analysis applied to Real Images

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The purpose of the project is to build a fast and reliable software for recognition of objects in aerial images. The preliminary stages consist of an edge detection by the analysis of a color gradient vector. After edge detection and feature extraction of the three-dimensional color histogram a set of parameters is extracted. The parameters taken into account to recognize regions are the histograms of the R, G, and B color components of the pixels and characteristics on the homogeneity and the shape. Principal Component Analysis was performed for examining relationships among several quantitative variables.The programmation language used operationally to translate is Java, Software Development Environment was Eclipse platform with Swing components for the Human Machine Interface. The software was applied to 400*400 pixels images. Real images have been tested to evaluate the performance concerning translation, rotation, zoom, and random noise addition. For reason of speed, the analysis can be made on only a part of the image, until 1/8 of the image.

References for the Project:

Hastie, T., Tibshirani, R., Friedman, J., 2001The Elements of Statistical Learning. Springer

Jaba, E., Qannari, A., 2013.Analyse Discriminante avec applications sous SPSS et SAS . Eyrolles, France.Translated for Editura Economica, Romania.

Busin, L., Vandenbroucke, N., Macaire, L., 2007. Color spaces and image segmentation.Internal report, UMR CNRS 8146, France.

Upward and downward statistical continuities

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A real valued function f defined on a subset E of $\mathbf{R}$, the set of real numbers, is statistically upward continuous if it preserves statistically upward half quasi-Cauchy sequences, is statistically downward continuous if it preserves statistically downward half quasi-Cauchy sequences, and a subset E of $\mathbf{R}$, is statistically upward compact if any sequence of points in E has a statistically upward half quasi-Cauchy subsequence, is statistically downward compact if any sequence of points in E has a statistically downward half quasi-Cauchy subsequence where a sequence (x_n) of points in $\mathbf{R}$ is called statistically upward half quasi-Cauchy if

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leq n : x_k - x_{k+1} \geq \varepsilon\}| = 0$$

is statistically downward half quasi-Cauchy if

$$\lim_{n\to\infty} \frac{1}{n}|\{k \leq n : x_{k+1} - x_k \geq \varepsilon\}| = 0$$

for every $\varepsilon > 0$. We investigate statistically upward continuity, statistically downward continuity, statistically upward half compactness, statistically downward half compactness and prove interesting theorems. It turns out that any statistically upward continuous function on a below bounded subset of $\mathbf{R}$ is uniformly continuous, and any statistically downward continuous function on an above bounded subset of $\mathbf{R}$ is uniformly continuous.

A Theorem for the Weighted Mean Method of Summability

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We investigate conditions needed for a weighted mean summable series to be convergent by using Kloosterman's method.The results of this paper generalize the well known results of Landau and Hardy.

PERT method with Monte Carlo Simulation, and the Most Probable Critical Path

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In this paper we use first the Monte Carlo simulation for PERT, in order to estimate the completion time of a project as the expectation of the simulated times, instead of solving a CPM problem using the expectations. In the same way, we will estimate the variances. Using the conditional probabilities, we will find the most probable critical path, and, conditioned by non-critical activity, the expectation and the variance of the time margin. For simulation we use both pre-defined distributions and bootstrap.

Application of Cox proportional hazard model in credit risk analysis

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There are many application of survival analysis in credit risk analysis, and one of the most important is estimating the client risk status in credit scoring through clients rating. In such models it is possible to include, not only the characteristics of a client and a credit product, but the macroeconomics factors, such as exchange rate and risk free rate. In this paper we present the application of Cox proportional hazard to the clients rating in process of approval for credit product. In forming Cox PH model, real data for development of the credit scoring model in financial institutions were used. Using the same data, we developed another model of the credit scoring, using logistic regression, in order to compare the models. It is shown that Cox PH approval model is a modern way of estimating the clients rating, which gives better results than approval model based on logistic regression.

Various types of spectra of operator matrices

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We describe various types of spectra (point, residual, continuous, essential) of upper triangular operator matrices on separable Hilbert spaces.

Generalizations of Granger Causality in Continuous Time and Some Types of Convergence

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In this paper we prove the invariance of some causality relationships between flows of information, represented by filtrations, under some types of convergence. We consider a statistical concept of causality which is based on Granger's definition of causality, but instead of time series we focus on continuous time processes. In addition, we give an alternative characterization of causality in continuous time.

Recent results on reverse order law

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Some recent results concerning various reverse order laws for the Moore-Penrose inverse of products of two and three Hilbert space operators are presented. Some identities between different reverse order laws are established, and Hartwig's theorem for triple reverse order law is generalized to the operator case.

Characterization of Some Classes of Operators

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We will present recent results on classes of compact and Fredholm operators between certain, new-defined, sequence spaces.

On operators on Hilbert C^*-modules

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We present some new results concerning bounded adjointable operators on Hilbert C*-modules.

Perturbed backward stochastic Volterra integral equations

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The paper discusses a large class of backward stochastic Volterra integral equations whose coefficients additively depend on small perturbations. Their solutions are compared in the L^2-sense, with the solutions of the appropriate unperturbed equations of the equal type. We prove that for an arbitrary $\eta > 0$ there exists an interval $[t(\eta), T]$ subset of [0,T] on which the L^2-difference between the solutions of perturbed and unperturbed equations is less than η. In contrast to similar problems about various perturbed forward and also backward stochastic differential equations, a completely different procedure must be applied on perturbed backward stochastic Volterra integral equations.

Spectrum of an bounded linear operator and invariant subspaces

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Let X_1 and X_2 be a closed invariant subspaces of a linearbounded operator $T \in B(X)$, where X is a Banach space. In the talk we will give conditions for invertibility of $T \in B(X)$ in respect of invertibility of itsrestriction to invariant subspaces in two cases:CASE I: $X = X_1 \oplus X_2$. In this case $\sigma(T) = \sigma(T_{|X_1}) \cup \sigma(T_{|X_2})$.CASE II: $X = X_1 + X_2$. In this case $X_1 \cap X_2 \neq \{0\}$ andthe relation of invertibility of T trough of invertibility of T_{X_1} and T_{X_2} start to be more complicate. We need to involve one more invariant subspace $X_1 \cap X_2$ and the restriction of T on it.

Some Tauberian Theorems For The Product Method Of Abel And Cesàro Summability to Convergence

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In this paper, we prove several new Tauberian theorems for the product of Abel and Cesáro summability methods which improve some classical Tauberian theorems for the Abel summability method.

Lacunary ward continuity in two normed spaces

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A function f defined on a subset E of a two normed space X is S_θ-ward (respectively, N_θ-ward) continuous if it preserves S_θ-quasi-Cauchy (respectively, N_θ-quasi-Cauchy) sequences of points in E, that is, a sequence $(f(x_k))$ is an S_θ-quasi-Cauchy (respectively, an N_θ-quasi-Cauchy) sequence whenever (x_k) is S_θ-quasi-Cauchy (respectively, N_θ-quasi-Cauchy). A subset E of X is S_θ-ward (respectively, N_θ-ward) compact if any sequence of points in E has an an S_θ-quasi-Cauchy (respectively, N_θ-quasi-Cauchy) subsequence. In this paper, not only S_θ-ward (respectively, N_θ-ward) continuity, but also some other kinds of continuities are investigated in two normed spaces. It turns out that uniform limit of S_θ-ward (respectively, N_θ-ward) continuous functions is again S_θ-ward (respectively, N_θ-ward) continuous.

On fixed point results for Matkowski type of mappings in G-metric spaces

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In 2006 Mustafa and Sims introduced the concept of G-metric spaces as a generalization of metric spaces. Some fixed point results for Matkowski type of mappings in G-metric spaces will be presented.

New examples of partial samples from the uniform AR(1) process and asymptotic distributions of extremes

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In this paper the joint limiting distribution of maximum of the specific sub-sample and maximum of the complete sample from the first order autoregressive process with uniform marginal distributions is obtained. There are considered several examples of partial samples, consisted of non-randomly selected terms of the full sample. It is well known that the uniform AR(1) process is strictly stationary random sequence, it doesn't satisfy condition of weak dependency, that prohibits clustering of extremes. As a consequence of this property, some interesting conclusions about joint asymptotic distributions are reached.

Derivations and local multipliers of C*-algebras

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We present some recent advances on a problem posed by GertPedersen back in 1978, which asks whether every derivation of aC*-algebra becomes inner in its local multiplier algebra.

Non commutative Müller regularity

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Both the joint spectrum of Joseph Taylor and the single variable spectrum of Tosio Kato arebased on the concept of exactness, leading to the idea of Muller regularity.

On explicit solutions of higher-order rational difference Equations and their systems

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Increasingly many actual problems arising in very different areas of nature and society are being reduced to difference equations or to systems of difference equations through mathematical modelling. In the last two decades there have been an huge number of papers and books devoted to qualitative analysis of solutions of nonlinear difference equations, including asymptotic behaviour, convergence, stability (both local and global), periodicity, asymptotic periodicity, attractivity, boundedness, etc. But, only rare number of them are devoted to explicit solving of nonlinear difference equations (i.e. in the so-called closed form). One of the reasons is the fact that it is not an easy problem, and in the most of the cases it is an unsolvable one. Nevertheless, the first step in qualitative analysis must be the patient check of the solvability of difference equation in closed form.In this paper, we shall try to solve some significant rational difference equations and their systems in closed form, whose solution has been missing in the literature so far.

Three unit system with repair and preventive maintenance

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We present a mathematical model of the three-unit system consisting of two active units and one reserve. For each unit, the time until the failure, the time until the preventive maintenance, the repair time and the preventive maintenance time are random variables, the first two are exponentially distributed and the last two have arbitrary probability distributions. The obtained system of equations is solved using the Laplace transform. From this it is possible to obtain the expected lifetime of the system. Under the assumption that the repair and the preventive maintenance are quick, we prove a limit theorem for the distribution of the lifetime of the system.

Quality control - history and recent development

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Statistical process control, or quality control, consists of wide range of methods based on monitoring the production process under study in order to detect any trend in the process. One can use the classical x-bar chart introduced by Shewhart in 1924, the R chart, or the CUSUM chart. We will give properties of these charts, their ARL (average run lenght) and OC (operating characteristic) curves, as well as Frechet chart, EWMA chart and some other. Time series analysis leads to another approach in quality control and one part of the talk will be devoted to these possibilities.

New limit theorems for the coupon collector's problem

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This paper deals with various extensions of the coupon collector's problem which are obtained by changing the stopping criterion. Proving limit results in these cases may involve a number of technical difficulties. We give some new limit theorems related to this problem, and discuss possible generalizations.

On composition of distributions

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Let F be a distribution and let f be a locallysummable function. The distribution $F(f)$ is defined as the neutrix limit of the sequence $\{F_n(f)\}$, where$F_n(x) = F(x) * \delta_n(x)$ and $\{\delta_n(x)\}$ is a certain sequence of infinitely differentiable functions convergingto the Dirac delta-function $\delta(x)$. Some composition of the distributions are proved to exists.

Results on product of distributions

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Modeling of singularities given by discontinuous functions or Schwartz distributions by means of Colombeau generalized functions has proved useful in many physical problems. In this paper results on product of distributions are derived. They are obtained in Colombeau differential algebra $\mathcal{G}(\mathbf{R})$ of generalized functions contains the space $\mathcal{D}'(\mathbf{R})$ of Schwartz distributions as a subspace, and has a notion of "association" that is a faithful generalization of the weak equality in $\mathcal{D}'(\mathbf{R})$.

Fixed point results in generalized metric spaces without Hausdorff property

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It is well-known that generalized metric spaces in the sense ofBranciari [A. Branciari, A fixed point theorem ofBanach-Caccioppoli type on a class of generalized metric spaces,Publ. Math. Debrecen 57 (2000) 31–37] might not be Hausdorff and, hence, there may exist sequences in them having more than one limit. Thus, in most of the fixed point results obtained recently in such spaces, Hausdorffness was additionally assumed. We show in this note that, nevertheless, most of these results remain valid without this assumption.

Some asymptotic properties of second order differential equation

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Asymptotic properties of solutions have been considered for some nonlinear second order dierential equations. The paper deals with investigation of bounded solutions, oscillatory solutions and another asymptotic properties. We also give sucient conditions, in order that the dierential equations of second order has innitely many solutions satisfying Cauchys problem. For generalinformation is referred a short reference.[1] Knezevic-Miljanovic, J, On asymptotical properties of solutions of second order non-linear equation,in Russian Uspehi matematicheskih nauk, T 47, 3(285), 1992, 163-164.[2] Kiguradze, I.T. and Chanturiya, T.A., Asymptotic Properties of Solutions of Nonautonomous Ordinary Dierential Equations (Asimptoticheskie svoistva reshenii neavtonomnykh obyknovennykh dierentsialnykh uravnenii), Moscow: Nauka, 1990.[3] J, Knezevic-Miljanovic, On Cauchy problem and solution of Emden Fowler type of equation, Dierential equations, Vol.45, N1, 1610-1612, 2009

Outer generalized inverses with prescribed idempotents of block matrices in Banach algebras

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We investigate additive results for (p,q)-outer generalized inverse of elements in Banach algebra, along with the representation of this inverse in a block matrix in the Banachiewicz-Schur form. Also, we give the representation of image-kernel (p,q)-outer generalized inverse of block matrices in rings. Additionally, we investigate the spectral properties of block matrices in a Banach algebra related to (p,q)-outer generalized inverse.

Constitutive equations with complex derivatives in viscoelastic models

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So far, models that describe wave propagation in viscoelastic media have been generalized in the fractional framework using real order fractional derivatives. In this talk we propose the use of complex order fractional derivatives for that purpose. We analyze stress-strain constitutive equations involving derivatives of complex order, and discuss mathematical and physical constraints that lead to acceptable models.This talk is based on joint work with Teodor M. Atanackovic, Stevan Pilipović and Dušan Zorica.

Non-autonomous stochastic Gilpin-Ayala competition model with time-dependent delay

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For considered model existence and uniqueness of the global positive solution is proven. Then, some other properties, such as boundedness, moment and pathwise estimation, extinction, non-persistence in time average and weak persistence, are established. Considered features are natural requirements from the biological point of view. Finally, examples with numerical simulations are given to illustrate our results.

Stability of stochastic vector-borne disease model with direct transmission

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In this paper we study stability of endemic equilibrium of stochastic vector-borne disease model with direct transmission. More precisely, we extend the deterministic epidemic model by introducing random perturbations around the endemic equilibrium state and obtain stability conditions for the considered model by suitable Lyapunov functions method. Finally, we provide real world example to illustrate results obtained through the paper.

Chaos Expansion Method for SDEs

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In this talk we consider classes of SDEs in which three main operators of Malliavin calculus appear, the Malliavin derivative, the Skorokhod integral and the Ornstein-Uhlenbeck operator. We apply the chaos expansion method in white noise spaces and obtain an explicit form of solutions of initial equations in the space of Kondratiev generalized stochastic processes. This talk is based on joint work with Stevan Pilipović and Dora Seleši.

Characteization of 2-inner product by strictly convex 2-norm of modul c

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Exploring and finding the necessary and sufficient conditions 2-normedspace be a 2-pre-Hilbert space, as a problem is focus of researching of manymathematicians. Some of the characterizations of 2-inner product are noted in [1], [4], [6] and [11]. In this paper we'll give the term strictly convex norm with positive module c,and will use that norm to do the characterization of 2-inner product.

Asymptotic analysis of positive solutions of second-order Emden-Fowler type differential equations in the framework of regular variation

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In this lecture we shall consider the second-order quasilinear differential equations

$$\text{(E)} \qquad \left(p(t)|x'|^{\alpha-1}x'\right)' + \omega\, q(t)|x|^{\beta-1}x = 0, \qquad \omega = \pm 1, \alpha > \beta > 0$$

where $p, q : [a, \infty) \to (0, \infty)$ are continuous functions. Asymptotic of nonoscilllatory solutions of (E) is essentially affected by the function $p(t)$, more precisely, by the integrals

$$P(t) = \int_a^t \frac{ds}{p(s)^{\frac{1}{\alpha}}} \qquad \text{or} \qquad \pi(t) = \int_t^\infty \frac{ds}{p(s)^{\frac{1}{\alpha}}},$$

in case $\int_a^\infty p(t)^{-1/\alpha} dt$ is either divergent or convergent. Therefore, to analyze the asymptotic of positive solutions,equations (E) is considered in the framework of the generalized Karamata functions with respect to $P(t)$ or $\pi(t)$. The purpose of this lecture to fully describe the overall structureof generalized regularly varying solutions with respect to $P(t)$ or $\pi(t)$,on the basis of behavior of coefficients $p(t)$ and $q(t)$ which are assumed to be generalized regularly varying functions. An application of the theory of regular variation gives the possibility of obtaining the necessary and sufficient conditions for the existence of three (four) possible types of positive solutions of eq. (E) with $\omega = 1$ ($\omega = -1$), together with the precise information about asymptotic behavior at infinity of all types of solutions.

Distortion of quasiregular mappings and equivalent norms on Lipschitz-type space

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We prove a quasiconformal analogue of Koebe's theorem related to the average Jacobian and use a normal family argument here to prove a quasiregular analogue of this result in certain domains in n-dimensional space.

As an application, we establish that Lipschitz-type properties are inherited by a quasiregular function from its modulu. We also prove some results of Hardy- Littlewood type for Lipschitz-type spaces in several dimensions, give the characterization of Lipschitz-type spaces for quasiquasiregular mappings by the average Jacobian and give a short review of the subject. In particular, we solve so called Dyakonov's problem.

Numerical Solution of the infinite-dimensional LQR/LQG-design problem

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The numerical treatment of linear quadratic regulator/gaussian design problems for parabolic partial differential equations requires solving large-scale Riccati equations. In the finite time horizon case, the differential Riccati equation (DRE) arises. We show that within a Galerkin projection framework the solutions of the finite-dimensional DREs converge in the strong operator topology to the solutions of the infinite-dimensional DREs. We also review efficient numerical methods for solving DREs capable of exploiting the structure on the problem (e.g. sparsity, symmetry or low-rank). We discuss several variants of the available methods, which allow to have a fast computation. In particular, the Rosenbrock type methods, BDF methods and different ways for solving the resulting algebraic Riccati equation. The performance of each of these methods is tested in numerical experiments. References[1] P. Benner, H. Mena: Rosenbrock methods for solving differential Riccati equations, IEEE Transactions on Automatic Control, to appear (accepted for publication April 8, 2013)[2] P. Benner, H. Mena: Numerical solution of the Infinite-Dimensional LQRProblem and the associated Differential Riccati Equations, MPI Magdeburg Preprint, MPIMD/12-13 (2012).

Some asymptotic properties of second order differential equation

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Asymptotic properties of solutions have been considered for some nonlinear second order differential equations. The paper deals with investigation of bounded solutions, oscillatory solutions and another asymptotic properties.We also give sufficient conditions, in order that the differential equations second order has infinitely many solutions satisfying Cauchys problem. For general information is referred a short reference.
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Gradient methods for computing the least-square solutions

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The idea of using two-point stepsize gradient methods for solving unconstrained minimization problems on $\mathbb{R}^n$ is applied on computing the least-squares solutions of a given linear system. Special attention is paid on corresponding modification of the scalar correction method introduced in [1]. Additionally, we consider the gradient iterative schemes as a useful tool for computing the Drazin-inverse solution of an appropriate linear system. The functionality of the exposed algorithms is based on a specific representation of the Drazin inverse solution, as well as the properties that we have studied [3].

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The Existence of the Density of the Ruin Time for the Sum of Two Compound Poisson Processes Perturbed by a Diffusion

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Let $(X_t, t \geq 0)$ be sum of a Brownian motion and a two independent compound Poisson processes and $\bar{\tau}_x$ the first hitting time of fixed level $x > 0$ by this stochastic process.We show existence of density with respect to Lebesgue measure. Link with ruin theory is also presented.

Intermediate solutions of fourth order quasilinear differential equations in the framework of regular variation

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Positive solutions of fourth-order quasilinear differential equation

$$(E) \qquad \big(p(t)|x''(t)|^{\alpha-1}\,x''(t)\big)'' + q(t)|x(t)|^{\beta-1}\,x(t) = 0, \quad \alpha > \beta > 0,$$

is studied under the assumption that functions $p(t)$ and $q(t)$ are positive, continuous functions satisfying conditions

$$\int_a^\infty \frac{t}{p(t)^{\frac{1}{\alpha}}}\,dt = \infty, \quad \int_a^\infty \left(\frac{t}{p(t)}\right)^{\frac{1}{\alpha}}\,dt = \infty\,.$$

The main objective is to discuss the existence and precise asymptotic behavior of intermediate solutions of (E). First, necessary and sufficient conditions for the existence of such solutions will be presented, and afterwards focusing attention to equation (E) with *generalized regularly varying coefficients* $p(t)$, $q(t)$ and to its *generalized regularly varying solution*, a complete information about the structure and the asymptotic behavior of positive solutions will be given, using Karamata's theory of regular variation.

Pantograph stochastic differential equations under nonlinear growth conditions and the Euler-Maruyama approximation

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In this paper pantograph stochastic differential equations are considered under nonlinear growth conditions. The existence, uniqueness and almost sure polynomial stability of solution is established. The whole consideration is affected by the presence of the unbounded delay in the arguments of coefficients of the equation of that type. Moreover, the convergence in probability of the appropriate Euler-Maruyama solution is proved under the same nonlinear growth conditions. Adding the linear growth condition, we show that the almost sure polynomial stability of the Euler-Maruyama solution implies the almost sure polynomial stability of the exact solution.

Bivariate Inflated-parameter Power Series Distributions

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The Inflated-parameter Power Series distributions (IPSD) was introduced in Minkova (2002), as a compound Power Series distributions (PSD) with geometric compounding distribution. The lack of memory property of the geometric distribution leads to some useful properties of the defined family of distributions. In these notes I introduce a bivariate version of the IPSDs. The new family of bivariate distributions is constructed by the trivariate reduction method. The probability mass function, recursion formulas and some properties are given. The particular cases of bivariate I-Poisson, I-binomial and I-negative binomial distributions are analyzed in detail. Minkova L.D. (2002). A generalization of the classical discrete distributions, Communications in Statistics: Theory and Methods, 31, 871-888.

Generalized Drazin inverse of block matrices in a Banach algebra

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Explicit representations of the generalized Drazin inverse of a block matrix having generalized Schur complement generalized Drazin invertible in Banach algebras are presented. Also we give equivalent conditions under which the group inverse of a block matrix exists and a formula for its computation. The provided results extend earlier works given in the literature.

On outperforming binomial thinning operator using geometric counting sequence in some real-life situations

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Some INAR models generated by the geometric counting sequence are presented and on the basis on their main features are compared to the INAR models based on the well known binomial thinning operator. Crucial properties of the observed models and their parameter estimators are considered. The main attentionis paid to the application of the models to the real data. Finally, on the observed kind of dynamical crime data, the possible reasons for outperforming binomial thinning based models by the corresponding ones generated by the negative binomial thinning operator, are discussed.

Fixed point theorems for non-self mappings with nonlinear contractive condition in strictly convex Menger PM-spaces

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In this paper we will define a notions of strictly convex and normal structure in Menger PM-space. Also, existence of a fixed point for non-self mappings with nonlinear contractive condition defined on strictly convex Menger PM-spaces will be proved. As a consequence of main result we will give probabilistic generalization of Assad and Kirk's result [Assad, N.A., Kirk, W.A., Fixed-point theorems for set-valued mappings of contractive type. Pacific J. Math. (43) 1972, 553—562.]

Pseudoinverses and reverse order law for matrices and operators

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We will consider reverse order law for matrices and opertarors and existance of various types of generalized inverses.

On Asymptotic Efficiency of Goodness of fit Tests for Pareto Distribution based on Characterizations

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In this paper we present some characterizations of Pareto distribution and construct some new goodness of fit tests based on them. We calculate their Bahadur efficiency for various alternatives and find locally optimal alternatives for those tests.

Partial metric fixed point theory: variations on a theorem of Fisher

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In 1994 S.G. Matthews introduced the concept of partial metric spaces as a generalization of that of metric spaces, in order to facilitate addressing some problems arising in the study of denotational semantics of dataflow networks and in the domain theory of computer science.

In this talk we present a generalization of a well known metric fixed point result of B. Fisher to the context of partial metric spaces.

Asymptotic Estimates of Solutions of a System of Delay Difference Equations with Continuous Time

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In this paper we study the asymptotic behaviour of solutions of the system of difference equations with continuous time

$$x(t) = A(t)x(t-1) + B(t)x(p(t)),$$

where $x(t)$ is an n-dimensional column vector, $A(t) = (a_{ij}(t))$, $B(t) = (b_{ij}(t))$ are $n \times n$ real matrix functions and the lag function $p(t)$ is a real function such that $p(t) < t$ and $\lim_{t\to\infty} p(t) = \infty$.We obtain asymptotic estimates of solutions of the considered system for the special cases when the lag function is between two known functions such as $p_1 t \leq p(t) \leq p_2 t$ for real numbers $0 < p_1 \leq p_2 < 1$, $\sqrt[p_2]{t} \leq p(t) \leq \sqrt[p_1]{t}$ for natural numbers $1 < p_1 \leq p_2$ and $p(t) = t - \delta(t)$, where $p_1 \leq \delta(t) \leq p_2$ for positive integers $1 \leq p_1 < p_2$.

Matrix domain of the triangle in l_p, $1 \leq p \leq \infty$

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The sequence spaces $C_0(\Delta_u^\lambda)$ and$C(\Delta_u^\lambda)$ have been recently introduced and studied. Inpresent paper, following the similar approach, we define a newsequence spaces $l_p(\Delta_u^\lambda)$, $1 \leq p \leq \infty$ andby applying the general methods as in [E.Malkowsky, V. Rakocević,On matrix domains of triangles, Appl. Math. Comput. 189 (2)(2007),1146-1163] we compute their β-duals, construct their basis andcharacterize some matrix classes concerning with these spaces. Wealso obtain estimates for the norms and the Hausdorff measures ofnoncompactness of the bounded linear operators L_A defined by theinfinity matrix $A \in (l_p(\Delta_u^\lambda), Y)$ where Y is one ofthe classic sequence spaces.

Convolution and Anti-Wick and Weyl quantization for ultradistributions

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Pseudo-differential operators with the global symbols of Shubin class act continuously on the space of tempered distributions. The extension of this class on the space of tempered ultradistributions of Beurling and Roumieu type was done by the use of the convolution on these spaces studied by the authors. Moreover, the Anti-Wick quantization of a standard symbol a equals to the Weyl quantization of a symbol b through the convolution of a and the gaussian kernel $e^{-|\cdot|^2}$ is extended to the new class of symbols. We will present, first, new results related to the convolution, second, the properties of the Anti-Wick and Weyl quantization for a new class of symbols and, third,the construction of the largest subspace of ultradistributions for which the convolution with the gaussian kernel exist.

Whittaker sampling

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Mean square and almost sure Whittaker-type derivative sampling theorems are obtained for the class $L^{\alpha}(\Omega, \mathfrak{F}, \mathsf{P})$, $0 \leq \alpha \leq 2$ of stochastic processes having spectral representation, with the aid of the Weierstraß σ. Functions of this class are represented by interpolatory series. The interpolation formulæ are interpreted in the α–mean and also in the almost sure P sense when the initial signal function and its derivatives (up to some fixed order) are sampled at the points of the integer lattice $\mathbb{Z}^2$. Finally, sampling sum convergence rate discussion is provided.

An Application of the ECF Method and Numerical Integration in Estimation of the Stochastic Volatility Models

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[1]The procedure of estimating parameters of Stochastic Volatility (SV) models, because of their specific structure, is much more complex than with the most similar nonlinear stochastic models. In this paper, the Empirical Characteristic Function (ECF) method is described in parameter estimations of the so-called standard SV model introduced by Taylor [4], as well as the original thresholds modification of this model, named the Split-SV model, introduced in [3]. The estimation procedure of the both of previously-mentioned models is based on minimization of the objective function which, in fact, represents the double integral with respect to the some weight function $g : \mathbb{R}^2 \to \mathbb{R}$. In our investigation we consider some typical, exponential classes of the weight functions $g(u_1, u_2)$. These exponential functions put more weight around the origin, which is in accordance to the fact that Characteristic Functions (CFs) contains the most of information around this point. On the other hand, an exponential weight function has the numerical advantage, because the objective function of the ECF method (i.e., the appropriate double integral) can be numerically approximated by using some of N-point cubature formulas. For this purpose, we use different types of cubature formulas, whose have been realized by authorized MATHEMATICA package `OrthogonalPolynomials` (see [1],[2]). Consequently, the objective function is minimized with respect to θ by a Nelder-Mead method, and estimation procedures are realized by the original authors' codes written in statistical programming language "R". Using these procedures, by different choices of weight functions, it is examined the performance of the ECF method, by statistical and numerical aspects. The numerical simulation of the obtained estimates is given, also. Finally, the standard SV model, and the Split-SV model as its alternative, are applied for fitting the empirical data: the daily returns of the exchange rates of GBP and USD per euro, and the efficiency of their fitting is compared.

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New first-order autoregressive model with applications

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The aim of this note is to introduce new AR(1) model with marginal Lindley ditsribution. It will be shown that distribution of the innovation sequence is mixture of the singular and absolute continuous distribution. Many statistical properties are derived such as: spectral density, some multi-step ahead conditional measures, run probabilities, stationary solution, uniqueness and ergodicity. The parameters are estimated using three different techniques, and their asymptotic distribution is examined. Some applications of the process are discussed to two real data sets and it is shown that the LAR(1) model fits better than other known non-Gaussian AR(1) models.

Bivariate autoregressive models in time series of counts forecasting

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Integer valued autoregressive (INAR) models have important part in time series of counts analysis. These models are composed of two components: survival process and innovation process. Survival component is based on thinning operator where the most used ones are binomial thinning and negative binomial thinning operators. In a situation where two time series are cross-correlated bivariate INAR models should be introduced. We present bivariate models based on binomial as well as negative binomial thinning operators. Innovation processes of these models are introduce in a manner to satisfy stationarity condition. Statistical properties of the models are discussed. Tests on a real data are conducted to demonstrate practical aspect of the models. Forecasting error is analyzed where special attention is payed on error made by survival component and error made by innovation component. Based on this residual analysis some aspects of future research are discussed.

Knowledge-based Recognition of Objects in Aerial Images

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Knowledge-based Recognition of Objects in Aerial Images Abdellah Qannari (*), Gerard Brunet (*), Mohamed Ibazizen (*)The project consists in determining regions in aerial images with use of statistical methods. The statistical methods are the Principal Component Analysis and the HAC (Hierarchical Ascending Classification). For this purpose, the preliminary stages consist of a fast and reliable edge detection, and a construction of regions with analysis of the three-dimensional color histogram. Principal Component Analysis was performed for examining relationships among several quantitative variables. The result of the edge pixel detection procedure is used for clustering-based segmentation in order to extract meaningful regions representing the objects in the scene. SAS (Statistical Analysis System) was used extensively and adaptations and specializations of some procedures were made with SAS/IML package. The software was applied to 400*400 pixels images .The performance has been evaluated with real images concerning translation, rotation, zoom, and random noise addition. A practical operationnal system has been implemented, with Java language. (*) University of Poitiers, Niort, France References for the Project Hastie, T., Tibshirani, R., Friedman, J., 2001The Elements of Statistical Learning. Springer Jaba, E., Qannari, A., 2013.Analyse Discriminante avec applications sous SPSS et SAS . Eyrolles, France.Translated for Editura Economica, Romania. Busin, L., Vandenbroucke, N., Macaire, L., 2007. Color spaces and image segmentation.Internal report, UMR CNRS 8146, France. Software Development EnvironmentSAS (Statistical Analysis System) development environment 9.3SAS IML Package (interactive Matrix Language)Eclipse Platform development environmentSwing Components graphical user interface-Matlab (interactive Matrix Laboratory), for preliminary steps

Quasihyponormal, EP and J-EP Matrices in Indefinite Inner Product Spaces

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We introduce a notion and give some properties of quasihyponormal matrices in spaces equipped with possibly degenerate indefinite inner product. We also present recent results concerning EP and J-EP matrices.

Geometry of an experiment and the function of expected values

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We consider the linear combinations of elements of two sequences: the first one a priory given nonnegative sequence and the second random sequence from the unit interval. We investigate the expected value of the smallest natural number such that the value of these linear combinations exceed a positive number. After geometrical considerations in multidimensional Euclidean spaces, we find the function which expresses the expected value. Especially, in the case of two pointed a priory given sequence, we will find close form function for the expected values. For all a priory numbers equal 1, it reduces to problem solved by B. Ćurgus and R.I. Jewetts in 2007. and solution to the exponential function.

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Endpoints of β-shrinking β-convergent multifunctions

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In this talk, we introduce β-shrinking, β-convergent β-generalized weak contractive multifunctions and give some results about the existence of endpoint of the multifunctions. We show that our main result generalize a recent related theorem.

Modified model of FitzHugh - Nagumo in neurodynamics

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Discusses some singularly perturbed system of ordinary differential equations with one fast and one slow variable, which is a modification of well-known FitzHugh - Nagumo of neurodynamics. The questions of the existence and the stability of non-classical relaxation cycle are examined.

On a Result of Uniqueness of Meromorphic Functions Sharing Two Sets with Less Cardinality

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Let f and g be two nonconstant meromorphic functions defined in the whole complex plane $\mathbb{C}$.Let S be a set of distinct elements of $\mathbb{C} \cup \{\infty\}$ and $E_f(S) = \bigcup_{a \in S}\{z : f(z) - a = 0\}$, where each zero is counted according to its multiplicity. In 1976F.Gross [2] raised the following question: Can one find finite setsS_j,$j = 1, 2$ such that any two nonconstant entire functions $f$and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must beidentical? To deal with the above question Lahiri[3] employed thenotion of weighted sharing and proved a theorem improving previous results. Very recently Banerjee-Majumder-Mukherjee[1] proved a theorem which improved the result of Lahiri as well as Banerjee. In this improvement however they used the set S shared by f and g with at least 6 elements. In this paper we reduce the cardinality of S to 4 and obtain their result under weaker condition.

Lacunary statistical convergence of double sequences in topological groups

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TBA

Conditions for the equivalence of power series and discrete power series methods of summability

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Discrete power series methods were introduced and their regularity results were developed by Watson. It was shown by Watson that discrete power series method (P_λ) strictly includes corresponding power series method (P). In the present work we present theorems showing when (P_λ) and (P) are equivalent methods and when two discrete power series methods are equivalent.

Some best possible Lipschitz constants for the distance ratio metric

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We study expansion/contraction properties of some common classesof mappings of the Euclidean space $\mathbb{R}^n, n \geq 2$, withrespect to the distance ratio metric. The first main case is thebehavior of Möbius transformations of the unit ball $\mathbb{B}^n$ in $\mathbb{R}^n$ onto itself. In the second main case westudy the behavior of bounded analytic functions or polynomials ofthe unit disk. In both cases sharp constants are obtained.For $n = 2$ we also propose some open problems.

Transformed confidence intervals for mean of skewed distributions

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Halls and Johnsons transformation methods for means of skewed distributions were studed here.

On the asymptotic behavior and expansion of the generalized

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We analyze the asymptotic behavior and expansion of the generalized Stieltjes transform, connected with some Abelian and Tauberian type theorems.

On the operational solutions of the higher order of fuzzy differential Equation

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We analyze the asymptotic behavior and expansion of the generalized Stieltjes transform, connected with some Abelian and Tauberian type theorems.

Some classical Tauberian theorems for the weighted mean methods of integrals

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Let $0 \neq p(x)$ be a nondecreasing real valued function on $[0,\infty)$ such that $p(0) = 0$. For a real valued function $f(x)$ which is continuous on $[0,\infty)$, we define

$$s(x) = \int_0^x f(t)dt, \tag{2}$$

and

$$\sigma_p(x) = \frac{1}{p(x)} \int_0^x p'(t)s(t)dt,$$

where $p'(t)$ is derivative of $p(t)$. If $\lim_{x\to\infty} \sigma_p(x) = s$ then the improper integral $\int_0^\infty f(t)dt$ is said to be weighted mean summable to a finite number s. It is known that if the limit $\lim_{x\to\infty} s(x) = s$ exists, then $\lim_{x\to\infty} \sigma_p(x) = s$ also exists. However, the converse is not always true.Adding some suitable conditions to weighted mean summability of $s(x)$ which are called Tauberian conditions may imply convergence of the integral (2). In this work, we give some Tauberian theorems to retrieve convergence of $s(x)$ out of weighted mean summability of $s(x)$ with some Tauberian conditions.

Some results on the class $\mathcal{U}$ and the class of $\alpha-$convex functions

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Let $\mathcal{A}$ denote the class of function $f(z)$ which are analytic in the unit disk $\mathbb{D} = \{z : |z| < 1\}$ with the normalization $f(0) = 0$ and $f'(0) = 1$.

Class $\mathcal{U}(\lambda, \mu)$, $\lambda > 0$ and $\mu \in \mathbb{C}$, is widely studied in recent years and is defined by

$$\mathcal{U}(\lambda,\mu) = \left\{ f \in \mathcal{A} : \frac{z}{f(z)} \neq 0 \text{ and } \left| \left(\frac{z}{f(z)} \right)^{1+\mu} \cdot f'(z) - 1 \right| < \lambda,\ z \in \mathbb{D} \right\}.$$

Further, the well known class of Janowski $\alpha-$convex functions is defined by

$$\mathcal{M}[A,B,\alpha] = \left\{ f \in \mathcal{A} : J(f,\alpha;z) \prec \frac{1+Az}{1+Bz} \right\},$$

where

$$J(f,\alpha;z) \equiv (1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right),$$

$\alpha \in \mathbb{R}$, $-1 \leq B$

Theory of statistical causality and extremality of measure

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Scientists have long been interested in problems of extremal laws. The relation between extremality and martingale representation property was discovered by Dellacherie, and was taken further andgiven its definite form by Jacod and Yor. Concept of statistical causality is closely connected to theextremality of measure. Namely, for weakly unique solutions of stochastic differential equations driven with semimartingales,can be established equivalence between the concept of causality and extremal measures of the solutions. The same can be proved for the solution of the martingale problem, too.

Economic statistical design of X bar control chart for non-normal symmetric distribution of quality characteristic

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In economic statistical design of a control chart, the economic-loss function is minimized subject to a constrained minimum value of power, maximum value of probability of false alarms and average time to signal an expected shift. This paper is concerned with the optimum economic statistical design of the X bar chart when quality characteristic has non-normal symmetric distribution. We considered three types of distributions: Student distribution, standard Laplace distribution and logistic distribution. For each of these distributions, we calculated theoretical distribution of standardized sample mean (or its best approximation) and approximated it with normal, Pearson VII and Johnson SU distributions. For considered example, constrained minimization of expected loss function was done using genetic algorithm in statistical software R. We compared results of economic statistical design of X-bar chart for theoretical distribution of standardized sample mean with the results for normal, Pearson and Johnson distributions. We found that, for all chosen distributions of quality characteristic, Pearson VII distribution and Johnson SU distribution give results very close to results based on theoretical distribution of standardized sample mean, while normal distribution gives much worse fit.

On asymptotic behaviour for a coupled system modeled by transverse vibrations of an inhomogeneous beam with a thermal effect

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We consider a flexible inhomogeneous beam of length Lclamped at both ends with a thermal effect. We prove thewell-posedness of the above model and analyze the behaviour of thesolution as $t \to +\infty$. The existence of solutions areproved using semigroup theory, and the stabilization of solutionsare obtained considering multiplier techniques.

Additive properties of the Drazin inverse for block matrix and its representations

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We present some properties of the Drazin inverse for the sum of two matrices. Also, some representations for the Drazin inverse of block matrix are given.

Condition Numbers for Moore-Penrose Inverse and Linear Least Squares

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Classical condition numbers are normwise: they measure the size of both input perturbations and output errors using some norms. To take into account the relative of each data component, componentwise condition numbers have been increasingly considered. These are mostly of two kinds: mixed and componentwise. In this talk, we give explicit expressions, computable from the data, for the mixed and componentwise condition numbers for the computation of the Moore-Penrose inverse as well as for the computation of solutions and residues of linear least squares problems. In both cases the data matrices have full column (row) rank.

Representations for the Drazin inverse of certain 2×2 block-operator matrices

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The Drazin inverse is a kind of generalized inverse which has avarious applications. In this talk we will focus on the representations for the Drazin inverse of certain 2x2 block operator matrices and the application of the Drazin inverse in the study of the second order homogeneous algebraic differential equations.

Coincidence points of two mappings acting in generalized metric spaces

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The properties of generalized metric spaces and covering mappings in these spaces are studied. Sufficient conditions for existence of coincidence points of covering and Lipschitz mappings acting in generalized matric spaces are obtained. These results are applied to investigate solvability of a certain type of functional equations.

On closed upper and lower semi-Browder operators

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We give several necessary and sufficient conditions for a closed operator to be upper (lower) semi-Browder. We also apply these results to give some characterizations of upper (lower) semi-Browder spectrum.

Field 4:

Geometry, Algebraic Geometry and Topology, Mathematical Physics

About cylindricity of Submanifolds with straight line in Minkowsky Space

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It is studied Finsler Submanifods in Minkowsky space,particularly, in Randers Space.It has been given generalization Toponogovs, Cheeger-Gromoll theorem for Randers Space. It has been proved sufficient conditions for cylindricity complete Finsler Submanifolds in Minkowsky Space. It has been found the conditions when from convexity of hypersurface in Randers space it follows positiveness of Flag curvature.

Some Symmetry Conditions of Almost S-manifolds

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In this paper, we consider almost S-manifolds. We study symmetries of these type manifolds, especially weak symmetries and Ricci symmetries. We get some conditions for non-existence weakly symmetric and weakly Ricci symmetric almost S-manifolds and we obtain a phi-symmetric and phi-Ricci symmetric almost S-manifolds is an etha-Einstein manifold under some conditions.

Invariant submanifolds of Hermitian bicontact structures (or normal metric contact pairs)

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We discuss some recent results concerning the invariant submanifolds of a normal metric contact pair (MCP) M with decomposable structure tensor ϕ (or Hermitian bicontact structure). By invariance of the submanifolds we mean invariance either by ϕ or by the two natural complex structures associated to M. In particular we prove that their minimality is related to the two Reeb vector fields associated to the MCP.

Harmonic connections and harmonic almost tangent structures with respect to natural metrics

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We study the harmonicity of certain geometric objects (such as (non)linear connections, almost complex, almost product, almost complex structures) with respect to the general natural metrics (and in particular natural diagonal metrics) on the tangent bundle.

Characteristic classes that cut equal areas and equal perimeters and prevent highly regular embeddings

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The properties of the regular representation bundles over the configuration space of k distinct points in the Euclidean space has classically been studied extensively by F. Cohen, Chisholm, V. Vassiliev, and many others.Motivated by geometric problems we present new computations of twisted Euler classes, Stiefel-Whitney classes and their monomials as well as corresponding Chern classes of these bundles.Thus, we not only extend and complete previous work, supplying for example a proof for a conjecture by Vassiliev, but also make progress in solving and extending variety of problems from Discrete Geometry, among them(i) the conjecture by Nandakumar and Ramana Rao that every convex polygon can be partitioned into n convex parts of equal area and perimeter,(ii) Borsuk's problem on the existence of "k-regular maps" between Euclidean spaces, which are required to map any k distinct points to k linearly independent vectors,(iii) Ghomi and Tabachnikov problem about the existence of "l-skew smooth embeddings" from a smooth manifold M to a Euclidean space E, which are required to map tangent spaces at l distinct points of M into l skew subspaces of E .(This lecture is based on joint work with Frederick Cohen, Wolfgang Lueck and Gunter M. Ziegler)

Curvature of Contact Metric Manifolds

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We will first review the basic ideas of contact manifolds and associated metrics. Then we will discuss a number of curvature results. Some of the questions will center on the sign of the curvature. We will then discuss certain curvature functionals over a compact contact manifold. As time permits other topics may include the action of the curvature operator on the Reeb vector field, local symmetry and conformal flatness, in the latter two cases extensions from the Sasakian case to general contact metric manifolds.

Lorentz geometry of 4-dimensional nilpotent Lie groups

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Geometric properties of Lie groups with left invariant Riemannian metric have been studied extensively. For example, the Milnor's classication of 3-dimensional Lie groups with left invariant Riemannian metric is classical reference. The geometry of Lie groups with left invariant pseudo-Riemannian metric is not so well known and with many open questions. Motivated by previously mentioned facts and our discussions with V. Matveev we investigate Lorentz geometry of 4-dimensional nilpotent Lie groups. First, we classify left invariant Lorentz metrics on 4-dimensional nilpotent Lie groups H3xR and G4. This research is motivated by results of Lauret where 3 and 4-dimensional Riemannian nilmanifolds are classified. As expected, we found much more metrics than in the Riemannian case. Note that Cordero and Parkersolved analogue problem for Lorentz 3-dimensional Lie groups. The geometry of these metrics is investigated. We calculate curvature tensor and holonomy groups of the metrics and investigate decomposability of the metrics. It is interesting that some of metrics have parallel null vector, but are not decomposable. These are exactly pp-wave metrics. Finally, we find projective classes of the metrics. We reveal that some ofmetrics are geometrically rigid, while others have projectively equivalent metrics that are also affinely equivalent. They are either decomposable metrics or indecomposable metrics with parallel null vector. It is interesting that the affinely equivalent metrics are also left invariant. We also check that in the Riemannian case (metrics from Lauret classication) all indecomposable metrics are geodesically rigid.

Almost geodesic mappings of manifolds with affine connection

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An almost geodesic mappings between torsion-free affinely connected manifolds an and ann were introduced and three types of these mappings: π_1, π_2 and π_3 were specified.For $n > 5$, this classification was proved to by complete. From these mappings was derived special almost geodesic mappings π_1^*.Interesting relations between these mappings it will be shown.

On generalized Ricci recurrent manifolds

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The object of the present talk is to introduce a new type of non flat Riemannian manifold called generalized Ricci recurrent manifolds. Some geometric properties have been studied. The existence of such a manifold is proved by several non-trivial examples. Finally, some applications of such a manifold in theory of relativity have been shown.

Geometry of Spin Manifolds

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We generalize the well known geometric characterizations of orientable n-dimensional manifolds, i.e., an n-dimensional closed PL manifold M is orientable if and only if each embedded circle in M has a regular neighborhood homeomorpic to the product of the circle with an $(n-1)$ ball. Theorem. An n-dimensional, orientable, closed PL manifold M, n>4, is spin if and only if each embedded closed surface F in M has a a regular neighborhood homeomorpic to the product of the surface F with an $(n-2)$ ball.

Operators on the ring of convex polytopes and the cd-index

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The ring of combinatorial convex polytopes developed in the works of Victor M. Buchstaber and the author gave new view on the classical problem of characterization of all the integral vectors that are vectors of flag numbers of convex polytopes. Many results and constructions in this area such as the Bayer-Billera equations, the toric g-polynomial, the cd-index obtained new interpretation that led to new results.Our aim is to build new operators on the ring of convex polytopes that have a nice geometric interpretation. The main result is the following.Let P be a convex polytope in $\boldsymbol{R}^n$ and l be a line through the origin in general position with respect to P. For any facet F of P consider the intersection x_F of l with the supporting hyperplane of F. For the top facet F_t and for the bottom facet F_b we have $x_F \in F$ and for all the others $x_F \notin F$. In this case consider the projection $\pi_F(F)$ of the facet F from the point x_F to the hyperplane through the origin orthogonal to l. Take the sum $K(P) = \sum_{F \neq F_t, F_b} \pi_F(F)$ in the ring of polytopes.The cd-index is a non-commutative polynomial in c and d, $\deg c = 1$, $\deg d = 2$, invented by Jonathan Fine. It captures all the information on the flag numbers of a polytope.**Theorem.** *For the cd-index Ξ we have $\Xi(P) = Ac + K(P)d$ for some non-commutative polynomial A. Moreover, $2A = Xi((d - CK)P)$, where dP is the sum of all the facets of P and $CP = 2\mathrm{pyr}(P) - \mathrm{bipyr}(P)$.* The first part of this result is obtained via polarity from the geometric interpretation of the cd-index by Carl Lee. The second part is obtained by the relations on the cd-index and the $C = 2\mathrm{pyr} - \mathrm{bipyr}$ and $D = \mathrm{bipyrpyr} - \mathrm{pyrbipyr}$ operations on the ring of polytopes.

Topological classification of integrable systems and billiards in confocal quadrics

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The theory of invariants based on the topological approach was suggested by A. T. Fomenko and developed by A.T.Fomenko, H.Zieschang, A. V. Bolsinov, and others for the study of integrable Hamiltonian systems with two degrees of freedom. This theory allows to investigate different qualitative properties of such systems and to conclude whether two systems are equivalent (in some sense) or not. Primarily, we mean the following three types of equivalence: Liouville equivalence, topological and smooth orbital equivalence. For each type of equivalence, a non-degenerate integrable system restricted to a 3-dimensional isoenergetic surface is assigned with a discrete invariant (molecule) which is a graph with some numerical marks. The main result of the theory can now be formulated in the following way: two integrable Hamiltonian systems considered on non-degenerate isoenergy 3-surfaces are equivalent (in one of the senses mentioned) if and only if the corresponding molecules are the same. In particular, two integrable non-degenerate systems are Liouville equivalent on 3-dimensional isoenergysufaces if and only of their Fomenko-Zieschanginvariants (graphs with numerical marks r,n,e) are the same.

V.Dragovich and M.Radnovich calculated these marks for some billiard systems in the 2-dimensional domain bounded by confocal quadrics. The work was continued by V.Fokicheva, who did the calculations of Fomenko-Zieschag invariants for so called "covering integrable billiards". The latter notion was introduced by A.Oshemkov and E.Kudryavtseva. Let us say, that the flat 2-dimensional domain Ω bounded by quadrics from the continuous family of quadrics (with parameter Λ) is called equivalent to the domain Ω' bounded by quadrics from the same family, iff Ω' is obtained from Ω by symmetries via axes and/or continuous change of parameter Λ with the only condition: Λ does not coincide with b.

Really, the flat billiard systems admit the following generalization. For example, consider k copies of the domain bounded by two confocal ellipses, and make a cut along the lower segment of the coordinate line O_y. Then glue cuts by the following rule: the left edge of the cut on the i-th copy is glued to the right edge of the cut on the i+1-th copy. This domain is called Δ_k . If we glue the rest of the edges of the cut together we get the new domain. If we consider generalization of this construction for another types on flat confocal billiards (in domains with angles), then we obtain a new large class of billards in "covering domains with angles". V.Fokicheva has obtained the topological classification of such domains and also the Liouville classification of corresponding Hamiltonian integrable billiard systems.

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Topology of Integrable Hamiltonian Systems - Recent Results

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The main goal of this talk is to demonstrate how the theory of invariants for integrable Hamiltonian systems with two degrees of freedom created by A.T. Fomenko, H. Zieschang, and A.V. Bolsinov helps to establish Liouville and orbital equivalence of some classical integrable systems. Three such systems are treated in the talk: the Euler case in rigid body dynamics, the Jacobi problem about geodesics on the ellipsoid and the Chaplygin case in dynamics of a rigid body in fluid. The first two systems were known to be Liouville and even topologically orbitally equivalent (Fomenko, Bolsinov). Now A.T.Fomenko and S.S.Nikolaenko (Moscow State University) show that the Chaplygin system is orbitally equivalent to the Euler and Jacobi systems. Main theorem. Let vCh (h) and vE (A, B, C, Ξ) be the Chaplygin and Euler systems (with parameters h and A,B,C, Ξ respectively) on regular 3-dimensional surfaces of constant energy Q = HCh = h and Q' = HE = Ξ. We suppose that the energy values h and Ξ belong to zones with the same number. Let vJ (a, b, c) denote the Jacobi system , i.e. geodesic flow on the ellipsoid (with parameters a,b,c), on a non-zero level of energy H. The triples of parameters (A, B, C) and (a, b, c) are viewed up to proportionality. Then the following statements hold.

1) If the energy value h of the Chaplygin system vCh (h) belongs to the first zone (1)Ch , then it is Liouville equivalent to the Euler system vE (A, B, C, Ξ) for any A, B, C and for any Ξ from the first zone (1)E . There also exists a one-parameter family of the Euler systems vE (A, B(A), C = 1, Ξ(A)) orbitally (topologically and smoothly) equivalent to vCh (h). This orbital equivalence can be extended on the four-dimensional neighbourhoods of the isoenergy surfaces. But for any A, B, C, Ξ the systems vCh (h) and vE (A, B, C, Ξ) are not topologically conjugate.

2) If the energy value h of the Chaplygin system vCh (h) belongs to the second zone (2)Ch , then it is Liouville equivalent to the Euler system vE (A, B, C, Ξ) for any A, B, C and for any Ξ from the second zone (2)E. There also exists a one-parameter family of the Euler systems vE (A, B(A), C = 1, Ξ(A)) topologically orbitally equivalent to vCh (h). But for any A, B, C, Ξ the systems vCh (h) and vE (A, B, C, Ξ) are not smoothly orbitally equivalent (even in the sense of C1 -smoothness) and are not topologically conjugate.

3) If the energy value h of the Chaplygin system vCh (h) belongs to the third zone (3)Ch , then it is Liouville equivalent to the Euler system vE (A, B, C) for any A, B, C (with the energy value from the third zone (3)E) and to the Jacobi system vJ (a, b, c) for any a, b, c. If h is large enough, there exist unique up to proportionality triples (A, B, C) and (a, b, c) such that the system vCh (h) is topologically orbitally equivalent to vE (A, B, C) and vJ (a, b, c). But this orbital equivalence cannot be made smooth (even in the sense of C1 -smoothness). Moreover, for any A, B, C and a, b, c the system vCh (Ξ) is not topologically conjugate with vE (A, B, C) or vJ (a, b, c).

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From simple Lie superalgebras to algebraic supergroups

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For any finite dimensional (complex) simple Lie superalgebra g, I present an explicit recipe to construct an algebraic supergroup G (in terms of its functor of points) whose tangent Lie superalgebra is the given g. This goes through a generalisation of the Chevalley's method.Chevalley's original procedure constructs a (semi)simple algebraic group starting from any complex, f. d. (semi)simple Lie algebra g and a faithful f. d. g-module V. The key tools to make use of is given by suitable integral forms - of g, U(g) and V - defined out of the notion of "Chevalley basis" for g.I shall show that this method can be succesfully adapted when one shifts from the classical setting to the "super" one, starting from a convenient notion of "Chevalley basis" for simple Lie superalgebras. Besides this existence result, I shall prove uniqueness: every connected algebraic supergroup whose Lie superalgebra be (f.d.) simple is isomorphic to one of the supergroups constructed via the Chevalley procedure. This eventually yields a complete classification of such supergroups.

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Some Remarks on Mathematical General Relativity Theory

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This talk will discuss some of the geometrical ideas behind Einstein's general theory of relativity. It will consider the foundations of the general theory and Einstein's attempt to build his gravitational field equations using the 4-dimensional space-time ideas generated by his special theory (as finalised by Minkowski), together with those facets of Newtonian gravitation theory which he wished to retain and the classical differential geometry developed by Riemann. Some simple, more modern developments will also be incorporated into the talk. These concern the possibilities for the field equations, Einstein's geodesic hypothesis regarding the motion of free particles in a gravitational field and the use of Killing symmetry in differential geometry to establish the theory of cosmology and wave theory.

On the distributions of the zeros of Alexander polynomials of knots

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We study various stabilities of Alexander polynomials of knots and links.In particular, we introduce some methods to construct knots such that the zeros of its Alexander polynomial are(i) modulus one, (i.e. circular stable),(ii) real, (i.e. real stable), or(iii) modulus one or real, (i.e. bi-stable).This is a joint work with K. Murasugi (Univ. of Toronto)

Equilibrium equations for plate theories in nonlinear elasticity

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This talk is about nonlinear bending theories of plates in fully nonlinear elasticity. This theory models, e.g., the bending behavior of paper. We will discuss the Euler-Lagrange equations and qualitative properties of critical points.

Meander knots and links

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Meanders are combinatorial objects with some topological properties of the interplay between planarity and connectedness, correspondingto the systems formed by the intersections of two curves in theplane, with equivalence up to homeomorphism within the plane. Theyoccur in polymer physics, algebraic geometry, mathematical theory ofmazes, and the study of planar algebras, especially theTemperley-Lieb algebra. We introduce concept of meander knots,2-component meander links and multi-component meander links andderive different families of meander knots and links from openmeanders with $n \leq 16$ crossings. We also define semi-meander knots(or knots with ordered Gauss code) and their product.

Topological invariants of Integrable Hamiltonian systems on the surfaces of revolution under the action of potential field

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Consider a manifold S, which is diffeomorphic to $(a, b) \times S$ (a and b are finite numbers), with metrics $ds^2 = dr^2 + f^2(r)d\varphi^2$ in polar coordinates $(r, \varphi(mod 2\pi))$. Functions $V(r)$, $f(r)$ are smooth functions on (a, b), and $f(r) > 0$ on (a, b). We call the function $V(r)$ a *potential function*, and $f(r)$ — a *function of revolution*. A system defined by pair of functions $(f(r), V(r))$ is an integrable Hamiltonian system. Let us call it a *system on the surface of revolution*.

Its phase space has a dimension of 4 (it has a coordinates $(r, \varphi, p_r, p_\varphi)$).

The system has two integrals of motion: H — the energy, and p_φ — the projection of momentum on axis of revolution. The Hamilton function of this system has a form

$$H = \frac{p_r^2}{2} + \frac{p_\varphi^2}{2f^2(r)} + V(r).$$

Definition. We call the map $\Phi : S \to \mathbb{R}^2 :$
$(r, \varphi, p_r, p_\varphi) \mapsto (H(r, \varphi, p_r, p_\varphi), p_\varphi(r, \varphi, p_r, p_\varphi))$ the *momentum map*.

Definition. If $rkd\Phi(x) < 2$, then x is called a *singular point*, and $\Phi(x)$ is called a *singular value*. The set Σ of singular points is called a *bifurcation diagram*.

Assume that we have constructed a bifurcation diagram for some system, i.e. we have a set of curves on the plane with coordinates (H, p_φ). Let us fix an arbitrary value of $H = H_0$. Then a level submanifold of the phase space, which corresponds to $H = H_0$, is a manifold of dimension 3, and it is called an *isoenergetic manifold* Q^3_H. Moreover, to any point (H_0, p_φ), which does not belong to the curves of bifurcation diagram, correspond one or more Liouville tori lying in Q^3_H, and to any point (H_0, p_φ), which belongs to the bifurcation curve, correspond some bifurcations of tori. The bifurcation of a certain type is called the *atom*.

For each value of H we can construct a graph with vertices corresponding to the bifurcations of tori and the ribs corresponding to the regular values of momentum map. This graph is called a *Fomenko–Ziechang invariant* (or simply a *molecule*).

Theorem 1. *If $f(r)$ and $V(r)$ are smooth functions on (a, b), then the system $(f(r), V(r))$ has only atoms of type A and B.*

Theorem 2. *All the marks of type "r" on the ribs of molecule have a value* 0 *or* ∞ *for investigated systems.*

Moreover, all Fomenko–Ziechang invariants were calculated (I will speak about it more detailly during my talk).

There exist another invariant — the invariant of trajectory equivalence. It is called a *function (and vector) of rotation* (it has not the same meaning that a function $f(r)$ has). You can read the theory about it in [1].

The functions of rotation for the investigated systems were calculated. And particularly was proved Fomenko hypothesis for the gravitational potential $V(r) = r$:

Fomenko hypothesis. *For two systems on the surface of revolution, given by the pair $(f(r), V(r))$, there exist a value $\widetilde{H}$, such that Fomenko–Ziechang invariants and vectors of rotation on its ribs coincide for any $H_1, H_2 : H_i > \widetilde{H}$, $i = 1, 2$.*

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Virtual Knot Theory

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This talk is an introduction to virtual knot theory and a discussion of invariants of virtual knots and virtual knot cobordism. Virtual knot theory is a generalization of classsical knot theory that has a diagrammatic system similar to the classical Reidemeister moves and studies embeddings of knots in thickened surfaces. Many fascinating phenomenal appear in this theory and it sheds light on the problem of whether the Jones polynomial detects the unknot.

Invariant distributions of compatible Poisson brackets

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It is well-known (see, for instance, [1], [2] and the references therein) that many integrable Hamiltonian systems arising in mechanics, geometry and mathematical physics are in fact bi-Hamiltonian. Recall that two Poisson brackets are called compatible if any their linear combination with constant coecients is also a Poisson bracket and that a dynamical system is called bi-Hamiltonian if it is Hamiltonian with respect to a pair of compatible Poisson brackets and all their non-trivial linear combinations.In [3] and [4] several new methods were described that allow to construct first integrals using the bi-Hamiltonian structure of a system and even prove integrability for many important classes of bi-Hamiltonian systems. Since it is interesting to nd out if there are any other ways to naturally associate a set of commuting functions to a pair of compatible Poisson brackets, we will study the following question: if there are any integrable distributions that can be described only in terms of the bi-Hamiltonian structure itself?In the talk we discuss the local structure of bi-Hamiltonian systems, including the classical Jordan-Kronecker theorem about the local structure of two bilinear form on a nite-dimensional vector space (see, for example, [5] and the references therein) and Turiel's theorem about the local structure of compatible symplectic structures in a neighborhood of a regular point (see [6]), and study the integrability of distributions which are invariant with respect to the group of local automorphisms for a pair of compatible Poisson brackets.

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The systems of the Kowalevski type and Magri's method of syzygies

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In a few recent papers a new approach to the Kowalevski integration procedure has been suggested. That approach gave a possibility to generalize the Kowalevski integration procedure and to apply it to a whole new class of systems, so called the systems of the Kowalevski type. The method is based on a class of polynomials – discriminantly separable polynomials, recently introduced by Dragović. Here, we investigate a relationship with an algorithm proposed by Franco Magri, so called Magri's method of syzygies. The results are joint with Vladimir Dragović.

Minimally non-Golod simplicial complexes in toric topology

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The notion of a Golod ring was introduced firstly in the work [4] for Noetherian local rings and is now a classical object of study in commutative algebra (homology of local rings). It appears in toric topology as a Stanley–Reisner ring (or face ring) $k[K]$ of a simplicial complex K over a ring of integers or a field of zero characteristic k. Due to Buchstaber and Panov theorem on the cohomology ring of a moment-angle complex $\mathcal{Z}_K$ [2] and the results of Berglund and Jollenbeck [1], it is just the case when multiplication in the ring $H^*(\mathcal{Z}_K, k)$ is trivial. For some special classes of simplicial complexes in [6] and [5] it was shown that their face rings $k[K]$ are Golod ones and in all those cases (if integer homology groups of all induced subcomplexes in K are torsion free) the corresponding moment-angle complexes have homotopy types of wedges of spheres.

In [1] the notion of a minimally non-Golod simplicial complex was introduced, that is $k[K]$ is not Golod itself but deleting of any vertex from K turns the face ring into a Golod one.

In my talk, based on [7], I will present some results to show that minimal non-Golodness of a face ring is in a close relation with the case when the simplicial complex is a boundary of a simplicial polytope. Moreover, for many of these polytopes (among them are the duals to vertex truncations of one or a product of two simplicies as well as even dimensional neighbourly polytopes, combinatorially different from simplices) a description of diffeomorphism types of the corresponding moment-angle manifolds is well known in toric topology [3]. These manifolds are connected sums of sphere products with two spheres in each product.

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On geodesic and holomorphically projective mappings

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We was proved that geodesic and holomorphically projective mappings of (pseudo-) Riemannian manifolds preserve the class of differentiability. Also, if the Einstein space admits a nontrivial geodesic mapping onto a (pseudo-) Riemannian manifold V, then V is an Einstein space. If a four-dimensional Einstein space with non-constant curvature globally admits a geodesic mapping f onto a (pseudo-) Riemannian manifold V with differentiability metric, then the mapping f is affine and, moreover, if the scalar curvature is non-vanishing, then the mapping is homothetic.

Singular Virasoro vectors: new explicit formulae and their applications

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We consider the Feigin-Fuchs-Wallach-Rocha-Carridi free resolution for computing the cohomology of the Lie algebra of polynomial vector fields on real line. This construction requires the explicit formulae for some families of Virasoro singular vectors. The formula for one family has been already obtained by Benoit-St-Aubin. The formula for another family of Virasoro singular vectors has been obtained and it gives the complete answer for the coboundary operator of the Feigin-Fuchs-Wallach-Rocha-Carridi free resolution.

Classical ball packing problems in the Thurston geometries

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(joint work with Jeno SZIRMAI)The famous KEPLER Conjecture on the densest packing the Euclidean 3-space E3 with equal balls, has recently been solved by Thomas HALES by computer. His procedure (in more than 200 pages) followed the strategy of László FEJES TÓTH. In this presentation we report the analogous problems in the other homogeneous geometries, the 8 THURSTON spaces: E3, S3, H3, $S2 \times R$, $H2 \times R$, SL2R, Nil and Sol. There many problems are open, e.g in János BOLYAI's hyperbolic space H3 as well. Our method is based on the intensive use of computer procedures, developed in recent works of the authors below.

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Monodromy groupoids of topological internal groupoids

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The notion of monodromy groupoid is used by J. Pradines togeneralise the standard construction of a simply connected Lie groupfrom a Lie algebra to a corresponding construction of a Lie groupoid-from a Lie algebroid. Let G be a topological groupoid such that the fibres of initial point map of the groupoid are path connectedand have universal covers. Let G be the disjoint union ofthe universal covers of the fibres at the base pointsidentities of the groupoid G. Then there is a groupoid structureon G defined by the concatenation composition of the pathsin the fibres. The topological groupoid structures of G are studied underin [3]. The groupoid G is called *monodromy groupoid* of G. A *group-groupoid* is a group object in the category of groupoids, equivalently, itis an internal category and hence an internal groupoid in the category of groups [6]. An alternative name, quite generally used, is "2-group". Recently the notion of monodromy for topological group-groupoids was introduced and investigated in [5].In this paper, the internal groupoid structure of monodromy groupoid for topological internal groupoids is developed.

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Infinitesimal bending influence on the Willmore energy of curves

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We describe the Willmore energy of embedded curves in three dimensional Euclidean space and examine the effect of infinitesimal bending of the curve on its energy.

On generalized null Mannheim curves in Minkowski space-time

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We define the generalized null Mannheim curves in Minkowski space-time. We obtain the necessary and sufficient conditions for the null Cartan curves to the the generalized null Mannheim curves in terms of their curvature functions and the curvature functions of their Mannheim partner curves. Finally, we give some examples of the generalized null Mannheim curves in Minkowski space-time.

The Chaplygin case in dynamics of a rigid body in fluid is orbitally equivalent to the Euler case and to the Jacobi problem

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We consider three classical integrable Hamiltonian systems: the Euler case in rigid body dynamics, the Jacobi problem about geodesics on the ellipsoid and the Chaplygin case in dynamics of a rigid body in fluid. It was proved by A. V. Bolsinov and A. T. Fomenko [1] that the first two systems are topologically orbitally equivalent, i.e. there exists a homeomorphism of the phase manifolds mapping the oriented trajectories of the first system to those of the second one. Now we prove that the Chaplygin system is orbitally equivalent to the Euler and Jacobi systems [2]. More precisely, the following statement holds.Theorem. For any "large" value of energy of the Chaplygin system there exists a unique Euler system and a unique Jacobi system topologically orbitally equivalent to the given Chaplygin system. For any other regular energy value of the Chaplygin system there exists a one-parameter family of the Euler systems topologically orbitally equivalent to the given Chaplygin system. In the case of "small" energies this orbital isomorphism is smooth.

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Od euklidske geometrije ka projektivnoj i natrag

Mileva Prvanovic
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Iako na prvi pogled mogu izgledati sasvim razlic(ito, mnoge teoreme euklidske geometrije su samo razne varijante jedne iste teoreme projektivne geometrije. Cilj ovog izlaganja je da to ilustruje na nekoliko primera.

Warping degree and ascending numbers of knots

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Warping degree and ascending number are two closely related knotinvariants defined by A. Kawauchi and M. Ozawa. Warping degree iscomputed for knots up to n=9 crossings by A. Shimizu, and we extend this computation to all knots up to n=12 crossings. Ascending numbers are computed by different authors for knots withat most n=8 crossings, and we determine exact ascending numbersfor 64 knots with at most n=10 crossings. By using the general formulae for the signature of knots, for 13 families of knots general formulae are obtained for their ascending numbers. The concepts of warping degree and ascending number we extend to pseudo knots and virtual knots.

Applications of the mean curvature flow associated to anisotropic Generalized Lagrange metrics in image processing

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The Geodesic Active Field (GAF) approach from image processing - whose mathematical background is the Riemannian theory of submanifolds, was recently extended by the authors to the Finslerian setting, for certain specific metrics of Randers type. The present work studies the significantly more flexible Generalized Lagrange (GL) extension, which allows a versatile adapting of the GAF process to Finslerian, pseudo-Finslerian and Lagrangian structures. The mathematically essential GAF mean curvature flow PDEs of three such GL structures (Randers-Ingarden, Synge-Beil and proper Generalized Lagrange) are explicitly obtained, discussed, implemented, and their corresponding feature evolution is compared with the classic results produced by the established original Riemannian GAF model.

On 3-triangulation of toroids

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Toroids (polyhedral torus) couldn't be convex, so it is questionable is it possible to 3-triangulate them (i.e. divide into tetrahedra). There will be discussed some examples of toroids and shown that for each $n \geq 9$ exists toroid for which triangulation is possible.

Volume conjecture, A-polynomial, Mahler measure and L-series

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The volume conjecture relates the asymptotics of colored Jones polynomial of a knot with the volume of knot complement. I will show how this is related to the A-polynomial of a knot, its Mahler measure and L-series of certain elliptic curve. Also, I will mention our recent results related to the quantized versions of some of these deep relationships.

Optimization Problems with Geometrical Graphs

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In this talk we present an overview on the modern state of optimal geometrical graphs investigation. A special attention is paid to various generalizations of the famous Steiner problem. In particular, we discuss recent results in new branch of the area mentioned above, namely, in minimal fillings theory. The latter extremals appeared as a mixture of Gromov minimal fillings (more exactly, of their stratified variant) and Steiner minimal trees.

On complexity of 3-manifolds with certain properties

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A complexity theory of 3-manifolds gives an useful approach for their classification. We will present modern results on explicit values and on bounds of complexity for infinite series of hyperbolic 3-manifolds which belong to the following classes: closed manifolds which are cyclic branched coverings of 2-bridge knots, manifolds with geoedesic boundary, and manifolds with cusps.

Lagrangian immersions of homogeneous 3-manifolds in complex space forms

Luc Vrancken

This is a report about joint work with X. Wang (Nankai University). We discuss Lagrangian submanifolds of the 3-dimensional complex space forms. In the last decades many results about Lagrangian submanifolds have been obtained characterising several classes of submanifolds. Some are valid in all dimensions, while other are only valid in dimension 2 or 3. Nevertheless, even in low dimensions, many open questions remain.

On the other hand, also in the last decades, many people have started to study the geometry of some special 3-dimensional homogeneous spaces. In particular many results have been obtained about how to study surfaces in such manifolds. Such manifolds can be divided into several classes depending of the dimension of the isometry group. They include

1. the spaces with 6-dimensional isometry group (Euclidean space, standard sphere S^3, hyperbolic space)
2. the spaces with 4-dimensional isometry group (which consists of $\mathbb{S}^2 \times \mathbb{R}$, $\mathbb{H}^2 \times \mathbb{R}$, the Berger spheres, the Heisenberg group Nil_3 and the universal covering of the Lie group $\mathrm{PSL}(2, \mathbb{R})$
3. the spaces with 3-dimensional isometry group (which include amongst others the Lie group Sol_3)

Of course equally interesting questions are whether any of these spaces can be isometrically immersed (locally or globally) as a Lagrangian submanifold in a complex projective space form, and whether such immersions, if they exist are necessarily unique.

A first result which can be formulated within this framework is the well known result by Ejiri.

Theorem 0.1. *Let ϕ be a minimal Lagrangian immersion from a 3-dimensional manifold with constant sectional curvature in the 3-dimensional complex space form. Then either*

- *M is totally geodesic, or*
- *M is congruent to a flat torus in the complex projective space.*

Note that without the assumption of minimality, the answer is not known. Even assuming that the immersion is a global one. There exist only some partial results, assuming additional assumptions.

If one looks at the other Thurston geometries (the Berger spheres, the Heisenberg group Nil_3, the universal covering of the Lie group $\mathrm{PSL}(2,\mathbb{R})$, the Lie group Sol_3, the product spaces $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$) the situatuion turns actually out to be more easy. Note that all these spaces are all quasi-Einstein, i.e. the Ricci tensor has an eigenvalue of multiplicity 2 at every point and have constant scalar curvature.

In contrast to immersions of real space forms, a Lagrangian isometric immersion (even locally) of one of the following homogeneous 3-manifolds (the Berger spheres, the Heisenberg group Nil_3, the universal covering of the Lie group $\mathrm{PSL}(2,\mathbb{R})$ and the Lie group Sol_3) in a complex space form is always minimal. Moreover the only possibility is a unique isometric Lagrangian immersion of a Berger sphere in $\mathbb{CP}^3$.

Theorem 0.2. *Let ϕ be a Lagrangian isometric immersion from (an open part of) one of the homogeneous 3-manifolds M (the Berger spheres, the Heisenberg group Nil_3, the universal covering of the Lie group $PSL(2,\mathbb{R})$ and the Lie group Sol_3) to a complex space form $\bar{M}^3(4c), c \in \{-1,0,1\}$. Then $c = 1$, M is a Berger sphere $\mathbb{S}^3_b(4/3,1)$, ϕ is minimal. And up to an isometry of $\mathbb{CP}^3$, the immersion ϕ is unique.*

For the other two types of homogeneous 3-manifolds (the product spaces $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$), in order to be able to reach a conclusion, as in the constant sectional curvature case, it is necassary to assume that the immersion is minimal. In that case, we have

Theorem 0.3. *Let ϕ be a minimal Lagrangian isometric immersion from an (open part of) $\mathbb{S}^2 \times \mathbb{R}$ to a 3-dimensional complex space form $\bar{M}^3(4c), c \in \{-1,0,1\}$. Then $c = 1$, ϕ is obtained as the Calabi product of the totally geodesic minimal immersion of $\mathbb{S}^2$ into $\mathbb{C}P^2$ and a point.*

Theorem 0.4. *There does not exist a minimal Lagrangian isometric immersion from an open part of $\mathbb{H}^2 \times \mathbb{R}$ into the 3-dimensional complex space form $\bar{M}^3(4c), c \in \{-1,0,1\}$.*

Pseudo-Riemannian geometry of Heisenberg group

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Geometry of Lie groups with left-invariant metrics is well known topic that dates back to classical Milnor's classication of 3-dimensional Lie groups with left-invariant, positive definite metric. The case of indefinite left-invariant metric is less known and with many open problems. The left-invariant metric is always considered up to equivalence with respect to the group of automorphisms. One of the striking differences between positive definite and indefinite setting is the result concerning 3-dimensional Heisenberg Lie group H3. There is, up to a homothety, the unique left-invariant positive definite metric on H3. In contrary, it is the result of Rahmani, that there exist three left-invariant Lorentzian metrics on the same group, one of which is flat. This result motivates us to study geometry of all nonequivalent definite and indefinite metrics on general Heisenberg group H2n+1.

Spacelike De Sitter Monge Forms in Sense of Slant Geometry

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(This talk is based on a joint work with Mikuri Asayama, Shyuichi Izumiya and Aiko Tamaoki.)It is known that there are three kinds of pseudo-spheres in Lorentz-Minkowski space which are called Hyperbolic space, de Sitter space and lightcone. A basic duality theorem for four Legendrian dualities related with these pseudo-spheres was obtained in [3]. These Legendrian dualities were extended in [4] depending on a parameter ϕ in $[0, \Pi/2]$. Moreover, as an application of these extended Legendrian dualities, one-parameter families of extrinsic differential geometries on spacelike hypersurfaces in de Sitter space were constructed in [2]. We call these geometries which include the results of [5] as a special case slant geometry of spacelike hypersurfaces in de Sitter space. In this talk, we first mention about the basic framework of slant geometry of spacelike hypersurfaces in de Sitter space. Then, we review the notion of spacelike de Sitter Monge form which was introduced in [5]. Finally, we give some examples of spacelike hypersurfaces in de Sitter space by means of spacelike de Sitter Monge forms in sense of slant geometry.

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Geometric properties of quantum completely integrable systems

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In the talk we are going to describe general geometric properties of algebraically integrable quantum completely integrable systems (i.e. supercomplete systems of commuting partial differential operators) of dimension two. In particular, we will give a detailed geometric description of spectral data for some old known systems.

Field 5:
History, Learning, Teaching of Mathematics and Informatics

Computational experiment approach to pre-college mathematics: Theory, pedagogy, tools

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The talk introduces the computational experiment approach to school mathematics curriculum by investigation a variety of mathematical models that were typically considered advanced in the pre-computer age. This approach makes it possible to connect sophisticated mathematical context and the modern day teaching practice. The talk will demonstrate how mathematical experimentation in the technological paradigm creates conditions for collateral learning to occur including the development of skills important for engineering applications of mathematics.

Use of simulated data and dynamic graphics for teaching statistics

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In teaching statistics to non statistics majors we face a problem of understanding of mathematical formalism. In applied statistics courses we usually use real life data related to the main subject matter of our students. Such data are interesting for students and motivate final interpretation of statistical results. For demonstration of statistical concepts, computer simulated data with known statistical properties can be used. The advantage of such data is that the results of analysis can be compared with known and pre-defined properties of data. Many important statistical concepts and procedures can be shown in an obvious way using computer simulations and dynamic graphics. Such simulations can sometimes be more convincing than proofs and are appreciated by students.

Student centered learning - what is it?

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Higher education was characterized by the lecture form during many decades. But, we are witnesses that with the intensive development of information technologies circumstances in this setting have been also intensively changed. The knowledge has changed. The students have also changed. They prefer instant information, they want to see their knowledge applied. Therefore system had to be changed through last 15-20 years - from teaching to educational system. This Education Reform involves:

- A shift from the lecture form to a student-centered approach to learning.
- A shift in the paradigm from measuring educational attainment to measuring of competencies.
- A change in emphasis from focus on content in curriculum to a problem solving in new situation.

In this communication we present our results done in this setting in cooperation with our colleagues and students from the University of Belgrade and the State University of Novi Pazar. More precisely, we analyze the role of Electronic teaching accessories, computers and software in the development of some competencies (spatial abilities, problem solving, critical way of thinking, etc.) as well as some soft skills within Geometry courses. Some of these results have obtained through international projects:

- Multimedia Technology for Mathematics and Computer Science Education, financially supported by DAAD, http://poincare.matf.bg.ac.rs/ daad/
- Geometry Education for Future Architects, financially supported by DAAD,http://www.math.tu-dresden.de/ lordick
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Invariant functions and n-to-1 maps

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We establish several examples of n-to-1 maps $f : \boldsymbol{R} \to \boldsymbol{R}$ satisfying the property that for every $y \in \boldsymbol{R}$ there exist exactly n real numbers $x_1, x_2, ..., x_n$ in the domain such that $f(x_i) = y, i = 1, 2, ..., n$. For odd n those examples can be found to be continuous (and differentiable), whereas for even n such examples are necessarily discontinuous. In connection with these type of functions, various invariant functions are considered, including even, odd, derivative invariant, stacked-periodic and $\omega-$invariant.

Mere geometrijskih objekata kao granične vrednosti u osnovnoj školi (problem usitnjavanja, nagomilavanja-utvrdjivanje granice nagomilavanja)

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Ideja ovog rada je da se učenicima u osnovnoj školi , predstavi opseg i površina kruga (i delovi kruga) preko poznatih pojmova upisivanjem i opisivanjem pravilnog mnogougla u isti krug. (Arhimedova ideja). Suprotno ideji da se koriste vrpca -kanap da bi se došlo do odnosa obima i prečnika kruga. Saglasno sa tom idejom da se učenicima u osnovnoj školi zapremina (obrazac) prave kružne kupe predstavi opisivanjem i upisivanjem pravilne piramida u i oko pomenute kupe . (slična ideja može da se uradi i za valjak) I na kraju, koristeći gore navedene ideje i koristeći sličnost trougla dočarati učenicima osnovne škole površinu lopte - sfere upisivanjem - opisivanjem u njoj i oko nje zarubljene kupe koje su nastale rotaciom pravilnih mnogouglova upisanaih - opisanih u i oko kruga (koji generiše loptu) koji rotira s tim mnogouglovima oko iste ose rotacije .

Korelacija matematike i ostalih nastavnih predmeta

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Korelacija - (ko+relacija) relacija je odnos, veza, kontakt, ko, kao predmetak u mnogim rijecima oznacava spajanje, pa se korelacija može definisati kao suodnos, uzajamna zavisnost, povezanost u harmonicnu cjelinu. U nastavi bi to znacilo neposredno povezivanje srodnih sadržaja iz razlicitih predmeta, kako bi nastava bila efikasnija, racionalnija i ekonomicnija. Obzirom da je matematika, kao nijedna druga nauka, prisutna u prirodi i društvu, to je korelacija u nastavi ovog predmeta imperativ i nužnost. Niko ne bi mogao odrediti, u slucaju nedostatka iste, ko je u vecem gubitku - sama matematika ili predmeti kao što su fizika, biologija, hemija, geografija...Prosto, oni ne bi mogli ni egzistirati bez matematike. Tacnije, mnoge nastavne sadržaje iz ovih predmeta ucenici ne bi mogli usvojiti bez prethodnog matematickog obrazovanja. (Navesti nekoliko lekcija iz ovih predmeta, kao primer, i što bi ucenici morali prethodno znati iz matematike da bi razumeli sadržaje). Zar je moguce uciti, posebno raditi zadatke iz hemije ne poznajuci proporciju? Zar je moguce razumeti geografske karte, ne poznavajuci razmeru? Kako razumeti nastavne sadržaje iz strucnih predmeta srednjih strucnih škola bez adekvatnog nivoa matematickog znanja? Covekova (ucenicka) misao djeluje na principu integrisanosti, odakle i težnja za sticanje celovitih znanja koje treba da pruža škola i da tako doprinosi razvoju razlicitih ucenikovih potencijala - prirodno-naucnih, umetnickih, socijalnih... Dakle, neophodnost zajednickog, tj. integrativnog planiranja nastave matematike sa ostalim predmetima, a posebno prirodno-naucnim, je prosto obavezujuca. Zbog toga, rad strucnog (ih) vijeca je jedan od bitnih faktora u obrazovanju. Ova aktivnost bi se morala, što prije, ugraditi, veoma znacajno, u obaveznu 42-casovnu nedeljnu radnu strukturu. Zevdžo Huric, prof.

Why is mathematics the language of science?

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The question "Is mathematics the language of science?" is hardly a polemical or a disputative issue: most mathematicians and scientists do concur that it is. In fact, history is replete with rather eloquent proclamations in favor of the assertion that mathematics is indeed a language, and in fact the language of science. Galileo's claim that, "Nature's great book is written in mathematical language," Sir James Jeans' proclamation in The Mysterious Universe that the Great Architect "... now begins to appear as a pure mathematician" (p. 165), Hogben's portrayal of mathematics as " ... the language of size, shape and order" and as "an essential part of the equipment of an intelligent citizen" (Hogben, 1936), are among the better known ones. More recent arguments include articles by Eckart (1984), Adler (1991), and Manin (2000).The more contentious and consequently the more interesting aspect of the question is why this is the case. In this paper, we claim that the answer is closely related to one's philosophical perspective and is based on the precept of construing mathematics as a continually transmuting complex system of communication.This paper will comprise three sections. In the first section, we will discuss the basics of social constructivism. We will follow this by a brief discussion of how social constructivism can be considered nominalist with respect to ontology. We will conclude our paper by showing that mathematics is the theory of form and structure that arises within language.

Metacognitive approach and its modifications in maths classes, primary school

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The important aspect for every student in acquiring the curriculum is the thinking of how to USE KNOWLEDGE that he has and gain confidence in solving the tasks. The aim of the work is to maintain how much the application of methodically created approach in maths classes, improves the quality of knowledge and the students' attitude towards maths, especially the aspect of metacognitive self-control. The research included the 3rd, 5th and 7th grade students, 147 of them (57 students of the 3rd grade, 21 students of the 5th grade, 69 students of the 7th grade). The methods of the written text and verbal discussion communication were used. The results of such research have shown that: 70,18

Problems that a teacher in primary school has while applying regular laws in work and the solutions that lead to better results in PISA tests and final exam

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In this work you can find practical solutions for postulates given in the rule book about the assessment in primary schools. The criteria for summative marks are not sufficiently defined in the regulations of evaluation, so they don't help us in fulfilling purposes and aims of the rule book itself. I have made this framework of assessment so that evaluation could be motivating and objective, moreover it should enable the continuous following of realizing the aims, outcomes and standards of achievements in students while acquiring the school programme. The organization of the school process with the student in the centre enables the teacher to record the elements of a formative mark as well as it enables the student to give answers and get better mark almost every class. As the pencil is not any more the basic means of intellectual work, the velocity of the personal computer is used to get the information about the abilities of every student, to choose the proper tasks, to record the student's progress and to inform parents and give them suggestions for further improvement in the shortest possible time interval. In order to use all this in my practice, I have made my own electronic workbook (book of problems) which contains precisely selected problems according to the level of achievement of educational standards, explorative problems and problems for students with special needs or inclusive programmes. The result of such approach is that parent becomes the real support to his child, the mark is the result of motivation not a "gift", and finally, every student knows how to acquire better mark. If a student wants to get a positive mark, it is necessary to achieve the elementary level of educational standards. That will help us make a big step in accomplishing the effort that the percentage of children with insufficient mathematic capabilities will be less than 15

Cognitive-visual approach to the teaching topic "Derivative of a function" by applying visualized problems

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The focus of the teaching topic "Derivative of a function" is both on the theoretical facts and problems on the functions and problems that are introduced in analytical form. However, in situations which are not only of analytical but also of graphic properties, it is affirmed that students are not able to apply their acquired knowledge. This discrepancy between the analytic and graphic, is also emphasized by the types of problems and requirements provided for students. They are largely only analytical. The way to overcome this discrepancy is to employ cognitive-visual approach to the study of derivative of a function applying visualized problems (problems which explicitly or implicitly include the image: in the very formulation of the problem, the way of solving it or in the end result) . This paper presents a new set of problems in which based on the assigned functions the properties of derivational functions are discussed, or vice versa. The precept of the problem comprises the one given in analytical records, and the other in the form of graphic. Problems visualized in this way enable the development and the use of visual thinking and the accomplisment of functional interdependence of graphic and analytical forms.

Šta prvo: Tales ili Pitagora

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U našem obrazovnom sistemu, kao i u mnogim drugim, odomaćila se tradicija da se Talesova teorema, povezana sa pojmom sličnosti trouglova, predaje posle Pitagorine teorema. U ovom saopštenju biće izloženo biše razloga za obrnuti pristup: Tales pre Pitagore.

Unapredjenje nastave matematike korišćenjem platforme "eZbirka"

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U ovom radu je prikazana platforma "eZbirka" - elektronska zbirka zadataka iz matematike za više razrede osnovne škole. Platforma je javno dostupna i kreirana je korišćenjem besplatnih alata (HTML5, PHP, MySQL, MatJax i Geogebra). eZbirka sadrži kvalitetne zadatke i može se koristiti kao pomoćno sredstvo za izvodjenje nastave u digitalnim kabinetima, organizovanje testova i zadavanje domaćih zadataka. Platforma je osmišljena tako da jasno definiše uloge nastavnika i učenika u procesu uçenja koje je potpomognuto modernim tehnološkim sredstvima. Omogućene funkcionalnosti sprečavaju učenike da prepisuju i podstiču ih na saradnju, a nastavnike upoznaju sa nivoom znanja učenika nakon svake nastavne jedinice. Povratna informacija koju nastavnici dobijaju predstavlja dobru osnovu za oblikovanje nastavnog procesa.

Učenje u nastavi matematike otkrivanjem

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Discovery learning is one of the most important teaching methods in teaching mathematics in grade school. This method develops independent research in the teaching process, because "running" the student teacher is an active factor in the educational process-that reveals the facts, rules, theorem proving and seeking ways to solve the problem assignments given for the problem of life. "Disclosure" as the act of learning in mathematics, with other modern methods-particularly the method of learning through problem solving, through concrete examples can be seen that it is very successful in terms of acquiring Self-education, self-assessment of students in mathematics. The largest research study conducted discovery in the field of teaching mathematics. This is not by chance, because the contents are most suitable for composing mathematics sequence detection and tracking tasks and problem-solving behavior of students in the process of search and discovery. A method is presented findings generally closely related to problem-oriented learning Here are shown examples of practical application of discovery learning methods in teaching mathematics in solving complex problem tasks in elementary school.

Matematički zadaci i razvoj matematičkog mišljenja

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Mathematics is a set of task specific data placed in a Osnos. This set contains data that are known and the unknown. From the basic data on the basis of relationships among data to determine the unknown value. Solving matemaičkih serves serves a variety of tasks specific objectives of teaching mathematics, without exaggeration, one could argue that the solution of tasks achieved (achieved) almost all the didactic aims of mathematics. By gaining a certain system of mathematical facts and ideas, as well as mastery of certain mathematical skills and habits, do not automatically mean the adequate development of mathematical thinking. Namely, in the teaching of mathematics, except for the formation of a certain "art" of thinking (ability to master a fixed operations and procedures), students need to be able to discover new connections and general procedures which enable them to solve new tasks and acquire new knowledge and skills. In brief, the students should form a general opinion procedures, and practices not only reviews the concrete situation. The characteristic of mathematical thinking can be viewed from several perspectives, such as:- Content or the type of reasoning (specifically, an abstract, intuitive, functional, dialectical, structural, creative or productive thinking, but the latter includes all the previous components), - General knowledge of methods and mathematical research (observation, induction, deduction, use analogies, mathematical modeling),- Form, that is determined by the quality of thought that style of thinking (flexibility, activity, orientation, cost, depth, breadth, criticality, originality, etc..), The subjective nature of personality traits (punctuality, perseverance, conciseness, concentration, curiosity, intellectual honesty, a tendency the creation, etc.).. The process of solving tasks, generally speaking you should have:The four main stages (phases): (1) Analysis and understanding of the task conditions (with schematic entry task),(2) Execution Plan (major step),(3) execution plan (in all its details),(4) review the task and its solution (test solution and the formulation of responses, analysis and feedback solutions, summary). Illustration of examples of pedagogical practice.

Primena različitih oblika individualizirane nastave matematike

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Individualized instruction. Not just a novelty ofour time, but the need of our time.This issue is devoted special attention in many countries, and latelyin our country.Diferencircijalne psiholofgije results and also theoretical andempirical research in pedagogy, didactics-discovered the existence ofsignificant individual differences among students.The basic tendency of individualized instruction is most ofspoosbnosti yet made to the maximum development of all students.The family there are different types of periodicals individualziraneteaching: Teaching papers, tasks of different difficulty levels,programmed learning, individualization of leisure activities,additional classes, remedial classes and d.Methods of individualized work is focused on to the students:-Learn to think independently and learn,-Work individually in smaller or larger groups,-Compare their results with other students,Maksimilno develop independence and acceptance of continuing new tasks.Each individualized work is independent work, students to build habitsfor life, work and study, both individual and collective.The requirements for the individualized-independent work, are usuallygiven in the form of teaching materials, worksheets, and can beconstituted in several ways.Tasks with two or more levels of difficulty: - -Preparation of individual sheets (cards) for each student requires long time, so resorts to drawing tasks in two or three difficulty levels,- Each task (leaf) has three variants: a lighter, harder and hardest.Within each group of tasks by the weight assigned tasks are graduallyand systematically (from mild, to severe, the logical structure).In the experiment with the application of this type of education (inthe experimental classes), students are divided into three groups,each group working independently varijatnu his performance while inthe control classes deal with the same tasks of medium difficulty.Ilustarcija practices. Ključne words: Individualization. Teaching. Methods. Shapes. Of applications.

Uloga problemskih zadataka iz oblasti kvadra i kocke na razvoj matematickih sposobnosti ucenika

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Nastava matematike ima veliku ulogu i znacaj u obrazovanju ne samo zato što pruža znanja potrebna radi primene u realnom životu, vec i zbog toga što snažno utice na razvoj matematickih sposobnosti ucenika. Ideja ovog rada je da odgovori na pitanje koje su to sposobnosti potrebne uceniku za uspešno rešavanje matematickih problema, a samim tim i bavljenje matematikom i da se pokaže uloga i znacaj rešavanja problemskih zadataka na razvoj matematickih sposobnosti ucenika. U ovom radu je izvršen izbor problemskih zadataka iz sadržaja nastave matematike za ucenike cetvrtog razreda osnovne škole u vezi sa kvadrom i kockom. Rešavanjem problemskih zadataka iz navedenih oblasti izvršen je uticaj na razvijanje sledecih sposobnosti ucenika: - apstraktno brojanje, - apstraktno sabiranje i množenje, - fluentnost ideja i rešenja, - posmatranje, uporedivanje i merenje i - logicko razmišljanje i zakljucivanje. Ovaj rad je deo eksperimentalnog programa realizovanog sa ucenicima cetvrtog razreda osnovnih škola u Leposavicu, Zvecanu i Kosovskoj Mitrovici. Primenom ovog programa iz oblasti kvadra i kocke izvršili smo proveru hipoteze koja se odnosi na trajnost i stabilnost stecenih znanja kao i na razumevanje matematickih svojstava i zakonitosti, te primene algoritama, cinjenica i informacija.

On The Existence of Mathematical Objects

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Continuous Fractions and Calculation of Pi

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Teaching Statistics on Faculties of Mathematics in Serbia

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Statistic thinking and talking is one of the important manners of contemporary communication. Statistical inference is included in almost all social and natural sciences. It will not be an exaggeration if we say even in all of them. In spite of variety of data that are the subject of statistical inference and teaching statistics within various faculties that are not devoted to mathematics itself, the fundamental method of the inference in statistics is mathematical.This talk introduces how or how much the statistics is studied on the faculties of mathematics in Serbia.

The development of teaching trigonometry as a paradigm for the development of mathematics teaching in Serbia in 19th century

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After independence, the Serbian authorities immediately began with the opening of schools. The teaching of mathematics received a special significance in the civil and military schools. The teaching of trigonometry from Atanasije Nikolić to Dimitrije Nešić goes way which extraordinary reflects the development of mathematics teaching.

The Origins and Beginnings of Probability Theory: Games of Chance, Problem of Points and Correspondence between Pascal and Fermat

Aleksandra Ravas

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The Origins and Beginnings of Probability Theory: Games of Chance, Problem of Points and Correspondence between Pascal and Fermat The aim of the author is to review how the human desire for winning in gambling, and in particular the Problem of Points, triggered the creation of a new branch in the field of mathematics. The history of games of chance is outlined, together with the early formulations and solutions of the Problem of Division of the Stakes. An examination of the correspondence between Pascal and Fermat which took place in 1654, considered today as the beginning of modern probability theory, follows.

What does exactly means "social implication of mathematics education"?

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Wirking group WG3 of the CERME 2(2001), among other things, was engage with the following question: How much are significant social and political encirclement for mathematics education realisation?

During session of the Thematic working group TG 12 of the CERME 3(2003) discourse was focused on the following questions:

- What kind of evidence can we provide to show that teacher education is worth the time and money invested in it?
- What kind of evidence do we not have but need to have in order to communicate with different people, like policy makers?
- What does evidence mean in the field of research on teacher education? What kind of evidence can we get in it?

In this presentation we will talk about conceptualization of social and political dimension of mathematics education in the Republic of Srpska.

Implementation of mathematical modeling on introducing the concept of the first derivative

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In this paper we present a way of introducing innovation in teaching process by applying new teaching method based on the principles of mathematical modeling. The paper deals with experiences from teaching process, concerning both aspects: teachers and students points of view. Direct application of new teaching method based on principles of mathematical modeling is illustrated in the paper by example of the teaching unit related to introducing the concept of the first derivative processed by using mathematical modeling as teaching method. Step by step procedure of making mathematical model is explained - from starting preparation, to implementation of the mathematical model and drawing conclusions from it. The positive effects of mathematical modeling to better understanding, the creation of advanced mathematical thinking, and the application of mathematical theory on solving real world problems are discussed and analyzed.

TIMSS matematika 2011

Dragana Stanojević

U radu je predstavljen program medjunarodnog TIMSS istraživanja. Navedene su opšte karakteristike i specifičnosti TIMSS testiranja. Prikazan je odnos važećeg nastavnog programa u Republici Srbiji i definisanog TIMSS programa za predmet Matematika. U radu su prikazani i nastavni programi iz matematike pet zemalja Singapura, Južne Koreje, Hon Konga, Tajpeja i Japana čiji su učenici imali najuspešnije rezultate na ovom medjunarodnom istraživanju. Takodje su u radu prikazana i postignuća učenika iz Srbije u zadacima koji pripadaju oblastima koji se ne izučavaju u mladjim razredima. Ovi rezultati mogu poslužiti za razvijanje i unapredjivanje postojećih nastavnih planova i programa, jer prikazuju u kom smeru se kreću učenička postignuća.

Extremely Complex Boolean Problems a Challenge for Mathematics and Informatics

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The theory of complexity allows us to classify given problems with regard to their complexity. We can be happy when the problem to be solved belongs to the class of logarithmic, linear, or polynomial complexity. But what can be done when the problem belongs to the most difficult exponential complexity? The number of function values of a Boolean function exponentially depends on the number of their variables. Hence, most of the Boolean problems have an exponential complexity depending on the number of variables.Within these classes the factual complexity of a problem can be measured by the required space to store all data, by the number of operations, or by the expected time to solve the problem. It is a challenge for mathematicians and computer scientists to solve extremely complex tasks where the basically required resources are beyond all realistic limits. The number of such extremely complex Boolean problem grows due to the progress in microelectronics as well as in science. The topic of this paper is to show how Mathematics and Informatics may successfully contribute to the solution of extremely complex Boolean problems.We take as example the edge coloring of complete bipartite graphs without complete monochromatic subgraphs K2,2. This task is equivalent to the rectangle-free coloring of grids using four colors. Before our recent solution it was an open question whether such colorings of the grids of the sizes 17x17, 17x18, 18x17, and 18x18 exist. The number of different 4-color patterns of the grid 18x18 is equal to $4^3$24 1.16798 $*$ $10^1$95. Due to this complexity it seemed to be hopeless to find solutions. Up to now, most powerful SAT-solvers were able to find solutions for subproblems with the size of approximately $10^1$35. Hence, it was our challenge to bridge the gap of $10^6$0. We summarize in this paper several approaches in order to gain the required knowledge to find the solution.

ICT in the contemporary teaching of Mathematics in high schools

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Technological equipment in schools in Serbia enables a concept of teaching process different from the traditional one. It allows an application of Information - Communication Technology, ICT.Since there are a number of experimental classes in secondary schools, where a teacher has a great freedom in the choice of teaching methods, forms and aids, there is a possibility to modernize the educational process by using computers.Previous knowledge of students in the field of computing enables the teachers to demand a very high standard, when using various application software and internet services.Modern teaching focuses is on students. Teachers are expected not only to be demonstrators, but also to educate students to use software packages for mathematics such as Mathematica and GeoGebra and services such as WolframAlpha.Moodle platform at the school site allows distance learning. This is the latest teaching form thus completing the entire teaching process.In this paper, we provide an overview of how this works in the school "Nikola Tesla" in Niš.

Design of Tests for Mathematics in the Moodle system

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Moodle is a learning management system for electronic education. One of the features in Moodle is the design of tests for automatically testing students' knowledge in a controlled environment. Automatic testing is not suitable for mathematics because a student has to implement a computational procedure that requires a prolonged thinking process for every answer . In this paper we discuss every aspect of creating tests for mathematics in Moodle such as the choice of types of questions, database size, number of questions per test, duration of the examination, giving negative points and others.

Z-generation is around us. What now?

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The generations of children born after in 1995. are called "Z" generation. These are the first generations who were born into a fully computerized world, and for them the virtual world is just as important as the physical one. They can not function without a computer, mobile phone, tablet, iPhone, iPad, and iPhone. The social skills of young members of this generation are quite different from ours. The usual generation gap between different groups (X, Y, Z) brings some misunderstanding and conflict. There is need to modernize the educational process. We should offer the contents that are watched and perceived rather than passively copied and listen, otherwise the students lose focus and interest. It is believed that the memorizing facts is unnecessary at the present time, when one can get all the information he or she needs in a few clicks. Therefore, the students need to work on the development of critical thinking, which will helps in filtering the multitude of available information. Teachers of the Z generation have to be open minded and computer literate to be able to make proper contact with their students.

Mathematical modelling in education

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The process of mathematical modelling (Gloria Stillman, 2012) is described with special references to its phases and their corresponding transitions. In particular the cognitive activities following the whole circle of mathematical modelling of real life problems are analyzed. Mathematical models are the elementary school contents.

Geogebra i matematicko-logicke igre

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Igra kao metod ucenja je karakteristicna i preovladujuca u predškolskom obrazovanju i vaspitanju. U manjem obimu ona se koristi i u nižim razredima osnovne škole, dok je na kasnijim uzrasnim nivoima gotovo iščezla. Savremene informacione tehnologije igri kao metodi ucenja daju novu dimenziju. Jedan od programskih paketa koji je u poslednje vreme stekao široku popularnost u nastavi matematike je Geogebra. Autor ovog saopštenja je na Naucno-strucnom skupu "Metodicki dani" 25. maja 2013. u Kikindi, predstavio mogucnosti Geogebre u kreiranju i distribuciji matematicko-didaktickih igara pomocu Geogebre za predškolce. U ovom radu ce biti predstavljeni primeri matematicko-logickih igara napravljenih u pomentom programskom paketu za ucenike starijih razreda osnovne škole i srednjoškolce. Ovakav vid prezentovanja nastavnih sadržaja podiže motivaciju za ucenje matematike i omogucava široku korelaciju matematickih sadržaja sa temama iz drugih nastavnih predmeta.

Field 6:

Theoretical Computer Sciences, Computer Sciences

Interconnection networks with recursive structure

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It is well known that some well-behaved graph topologies are important classes of interconnection networks in parallel and distributed computing. We propose some interconnection models based on the recursive structure for multicomputer networks. In addition, comparative survey study of the properties of different recursive models are presented.

Algorithms for determinization of fuzzy and weighted automata

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We provide various algorithms for determinization of fuzzy automata over complete residuated lattices: construction of the Nerode automataon, determinization by means of right invariant fuzzy quasi-orders, construction of the children automaton, determinization by means of closures, and Brzozowski's type determinization. We compare them with respect to the size of the automaton that they produce and their computational time. We transfer some of these algorithms to weighted automata over semirings and strong bimonoids.

Genetic Algorithm and Local Search Hybridization to Solve Traveling Salesman Problem

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In this study, a new hybridization of GA and local search based on a new similarity-based control mechanism is proposed and its behavior on different TSP instances is compared with simple GA. The experimental results show that the proposed hybrid algorithm yields the optimal tour length in most of the cases, especially in the TSP instances with higher complexity.

Algorithms for computing the greatest simulations and bisimulations for fuzzy automata

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Two types of simulations (forward and backward simulations) and four types of bisimulations (forward, backward, forward–backward, and backward–forward bisimulations) between fuzzy automata have been introduced by ćirić et al. (2012). If there is at least one simulation/bisimulation of some of these types between the given fuzzy automata, it has been proved that there is the greatest simulation/bisimulation of this kind. In the present work, for any of the above-mentioned types of simulations/bisimulations we provide an efficient algorithm for deciding whether there is a simulation/bisimulation of this type between the given fuzzy automata, and for computing the greatest one, whenever it exists. The algorithms are based on the method developed by Ignjatović et al. (2010), which comes down to the computing of the greatest post-fixed point, contained in a given fuzzy relation, of an isotone function on the lattice of fuzzy relations. The corresponding algorithms are also provided for nondeterministic automata.

State reduction for fuzzy automata with outputs

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We provide algorithms for state reduction of fuzzy automata with output. We show that the state reduction problem for these automata boils down to the problem of finding solutions to certain systems of fuzzy relation equations and we give procedures for computing the greatest solutions to these systems.

WebGRAPH - an online graph filtering system

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WebGRAPH is an online graph filtering system based on newGRAPH API. It enables user to filter catalogues of graphs withrespect to given conditions. Catalogues of graphs can be either uploaded (in g6 format) or they can be chosen from agiven database of small graphs. We've developed a simple language for defining the filtering conditions. Filteringconditions include standard simple graph invariants, as well as complex invariants (i.e. graph spectrum and other) thatcan be computed by newGRAPH (http://www.mi.sanu.ac.rs/newgraph/)

Construction and verification of the spherical coverage by hypercaps

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In this talk we consider the coverage of d-dimensional unit hypersphere by the spherical caps. Two main problems are discussed.In the first problem, one has a given set of caps and needs to check whether the complete hypersphere is covered or not. It is proven that such problem is NP-hard and the recursive algorithm is presented.The other problem is the construction of the covering set consisting of N identical caps with the minimal angle. This problem have direct applications in covering the globe by N different transmitters and/or satellites. Different construction methods are introduced and the results are discussed.

Partial volume simulation in software breast phantoms

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We discuss novel algorithm for simulation of voxels containing multiple materials in software breast phantoms. A modification to our previous simulation of breast anatomy is proposed to improve the quality of simulated projections . Previously, we developed a novel recursive partitioning algorithm for breast anatomy simulation in which each phantom voxel was assumed to contain a single tissue type. As a result, phantom projection images displayed notable artifacts near the borders between regions of different materials, particularly at the skin-air boundary. We provide theoretical evidence that improvement in image quality without reducing voxel size is achievable by the simulation of partial volume averaging. Thus, voxels containing more than one simulated tissue type are allowed. The linear x-ray attenuation coefficient of the voxels is the sum of the linear attenuation coefficients weighted by the voxel subvolume occupied by each tissue type. To calculate sub volumes, a local planar approximation of the boundary surface is employed. In the two-material case, the subvolumes in each voxel are computed by decomposition into simple geometric shapes. In the three-material case, by application of the Gauss-Ostrogradsky theorem, the 3D subvolume problem is converted into one of a few simpler 2D surface area problems. An efficient encoding scheme is proposed for the type and proportion of simulated tissues in each voxel. Statistical methodology is introduced to validate the implementation of the algorithm and evaluate the quantitative error of the approximation. We illustrate the proposed methodology on phantom slices and simulated mammographic projections. Our results indicate that the proposed simulation has improved image quality by reducing quantization artifacts.

Mathematical Issues in Software Breast Phantom Simulation

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Anthropomorphic breast phantoms are used for preclinical testing or optimization of imaging systems or image analysis methods. We discuss mathematical issues of recursive partitioning algorithms for simulation of software breast phantoms. The considered aspects include modeling of tissues and anatomical details (e.g., Cooper's ligaments, milk ducts) and control and validation of model parameters. We demonstrate the asymptotic time complexity of the algorithm. We provide theoretical evidence of possible improvement in projected image quality without reducing voxel size. We propose characterization of multi-tissue voxels through calculation of subvolumes occupied by each tissue type. To calculate subvolumes we utilize local planar approximations of the boundary surfaces and decomposition into simple geometric shapes. Statistical methodology is introduced to validate the implementation of the algorithm and evaluate the quantitative error of the approximation.

Toward The Formalization of Software Measurement by Involving Network Theory

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Complex network theory is based on graph theory and statistical analysis Complex real-world systems represented by typed and/or attributed graphs form different kinds of complex networks. Statistical methods applied on these graphs provide powerful mechanism in network analysis. Complex networks theory has an application in many areas such are social networks, computer networks, etc. In a context of software engineering and software development we can talk about special type of complex networks – software networks. Software networks are directed graphs representing relationships between software entities (packages, classes, modules, methods, functions, procedures, etc.). Software network can be observed as a static representation of software code and design and can be used in analysis of the quality of software development process and software product with particular application in a field of large-scale software systems. Software metrics are basic mechanism in software quality analysis. Software metric can be defined as measure that reflects some property of a software product or its specification. Software metrics can be calculated based on a static representation of source code and design in which case they become part of static analysis. The aim of this paper is to investigate possibility of application of complex networks theory in static analysis of software quality. Basic goal is to set relations between particular software metrics and corresponding statistical measures from complex networks theory. This would be a basis for stronger formalisation of software metrics and establishing stronger ties between these two fields that have a common goal. Furthermore, this paper will provide an overview of available approaches in this direction.

A new efficient algorithm for finding k shortest simple paths in undirected graphs

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We present a new algorithm for enumerating the k shortest simple (loopless) paths in nondecreasing order between a designated pair of vertices in a given undirected graph. Our algorithm is based on A^* algorithm and uses Dijkstra's algorithm to estimate the cost to rich the destination node. The algorithm calls Dijkstra's algorithm only ones and uses obtained values for generating candidates for next path. It also uses efficient method for detecting loops in subpaths to early prune inadmissible paths with loops. Some experimental results for different types of graphs are provided to illustrate the efficiency and applicability of the algorithm.

The construction of co-authorship networks supported by author name analysis

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Co-authorship networks are undirected graphs of collaboration between scientists. Analysis of co-authorship networks can help us to quantify and understand the structure and evolution of collaborations in academic societies. Additionally, patterns mined from co-authorship networks have a few important applications such as academic author ranking, reviewer recommendation and prediction of future research collaborations. The nodes in a co-authorship network are identified by researcher names contained in the bibliographical records used as the data source in network extraction. For manually created bibliographical databases there might be spelling errors and other inconsistencies in bibliographical records. In this paper, we present a semi-automatic technique for the construction of co-authorship networks that involves analysis of author names. Namely, different string similarity measures are employed to detect authors whose names appear differently in database records. The technique is experimentally evaluated on bibliographical records of the electronic library of the Mathematical Institute of the Serbian Academy of Science and Arts (eLib). In the paper we discuss the differences in both macroscopic and microscopic properties of the eLib co-authorship networks obtained with and without name analysis.

Rank of fuzzy matrices. Applications in state reduction of fuzzy automata

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In this talk different types of ranks of fuzzy matrices will be considered. In particular, ranks of fuzzy matrices representing fuzzy quasi-orders are studied in detail. It will be shown that fuzzy matrix decomposition by ranks can be used in state reduction of fuzzy automata.

Application of fuzzy relation equations in data analysis

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This paper discusses certain systems of fuzzy relation equations with one or two unknown fuzzy relations. It is shown that these systems have the greatest solutions, which are either fuzzy equivalences or fuzzy quasi-orders, or pairs of such fuzzy relations, and are provided efficient algorithms for computing these greatest solutions. It is also shown how to use these fuzzy equivalences and fuzzy quasi-orders in the reduction of data represented by Boolean and fuzzy data tables.

Adaptive Clustering of Gaussian Mixture components in Nonlinear Bayesian Estimation of Recurrent Neural Networks

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In this paper we have considered training of recurrent neural network as nonlinear Bayesian estimation. The nonlinear Bayesian estimator is implemented as Gaussian Sum filter. The major drawback of the proposed algorithm is exponential growth of the number of components in the posterior density of the recurrent neural networks state vector. In order to prevent exponential explosion, we have implemented an adaptive clustering algorithm which clusters the components of the mixture and replaces each cluster with a single Gaussian.

Syntax-directed models for statistical machine translation

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Formal methods from the theory of weighted tree automata and tree transducers have received an increasing attention in the area of statistical machine translation. In this talk I would like to illustrate one aspect of their usefulness.

Representation of computations in Knot theory by using XML technologies

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One of the most convenient way to represent different mathematical results is based on XML applications. We use MathML and SVG to represent some results from Knot Theory received by using Wolfram's Mathematica. The information obtained in this way can be easily distributed and processed. Some examples of knot operations and invariants are presented. The advantages of using XML technologies in this field are pointed out.

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