Workshop on Generalized Inverse and its Applications

Invited Speakers

Program

Abstract

Southeast University, Nanjing

November 2-4, 2012

Invited Speakers:

Changjiang Bu, College of Science, Harbin Engineering University, China, buchangjiang@hrbeu.edu.cn

D.S. Cvetković-Ilić, Faculty of Sciences and Mathematics, University of Niš, Serbia, dragana@pmf.ni.ac.rs

Jianlong Chen, Department of Mathematics, Southeast University, China, jlchen@seu.edu.cn

N. Castro-González, Facultad de Informática, Universidad Politécnica de Madrid, Spain, nieves@fi.upm.es

J. Ljubisavljević, Faculty of Medicine, University of Niš, Serbia, jelena.ljubisavljevic@medfak.ni.ac.rs

Xiaoji Liu, School of Science, Guangxi University for Nationalities, China, xiaojiliu72@yahoo.com.cn

Jipu Ma, Nanjing University, Tseng Yuan-Rong Functional Research Center, Harbin Normal University, China, jipuma@126.com

Yongge Tian, China Economics and Management Academy, Central University of Finance and Economics, China, yongge.tian@gmail.com

Yimin Wei, School of Mathematical Sciences, Fudan University, China, ymwei@fudan.edu.cn

Qingxiang Xu, Department of Mathematics, Shanghai Normal University, China, qxxu67@sit.edu.cn

Bing Zheng, School of Mathematics and Statistics, Lanzhou University, China, bzheng@lzu.edu.cn

Organizing Committee:

Jianlong Chen, Department of Mathematics, Southeast University, China, jlchen@seu.edu.cn

Yimin Wei, School of Mathematical Sciences, Fudan University, China, ymwei@fudan.edu.cn

Program

November 3, 2012 (Saturday)

Time	
08:30--09:00	Opening ceremony、Photo-taking
	Chair: Yimin Wei
09:00--09:30	Jipu Ma (Nanjing University,Tseng Yuan-Rong Functional Research Center, Harbin Normal University) Some applications of generalized inverses
09:35--10:05	N. Castro- González (Universidad Politécnica de Madrid, Spain) Multiplicative perturbation for the Moore-Penrose inverse and accurate solution of least squares problems
10:10--10:40	Yongge Tian (Central University of Finance and Economics) Using generalized inverses of matrices to establish closed-form solutions of LMIs
10:40--10:55	Tea Break
	Chair: Jianlong Chen
10:55--11:25	D.S. Cvetković-Ilić (University of Niš, Serbia) Some additive properties of the Drazin inverse and (2, 2, 0) Drazin problem
11:30--12:00	J. Ljubisavljević (University of Niš, Serbia) The additivity of the Drazin inverse for block matrices
12:00--13:45	Lunch (Liu Yuan Hotel)

	Chair: N. Castro-González
14:00--14:30	Yimin Wei (Fudan University) Solving singular linear equations and generalized inverses
14:35--15:05	Changjiang Bu (Harbin Engineering University) Generalized inverse and combinatorial matrix theory
15:10--15:40	Xiaoji Liu (Guangxi University for Nationalities) Simultaneous decomposition of two EP matrices with applications
15:40--15:55	Tea Break
	Chair: D.S. Cvetković-Ilić
15:55--16:25	Qingxiang Xu (Shanghai Institute of Technology, Shanghai Normal University) Solvability of certain quadratic operator equations and representations of Drazin inverses
16:30--17:00	Bing Zheng (Lanzhou University) The multiplicative perturbation bounds of the group inverse and oblique projection
17:05--17:35	Jianlong Chen (Southeast University) Drazin inverse and generalized (pseudo)Drazin inverse in a ring
18:00--20:00	Banquet (Liu Yuan Hotel)

November 4, 2012 (Sunday)

08:00--15:00	Tours: Presidential palace, Dr Sun Yet-Sen's Mausoleum, Confucius Temple, Zhonghua Gate

Abstract

Generalized inverse and combinatorial matrix theory

Changjiang Bu

College of Science, Harbin Engineering University
E-mail: buchangjiang@hrbeu.edu.cn

In the report, matrices with signed generalized inverse, resistance distance and group inverse of Laplacian matrix are discussed.

Some additive properties of the Drazin inverse and (2, 2, 0) Drazin problem

D.S. Cvetković-Ilić

Faculty of Sciences and Mathematics University of Niš, Serbia
E-mail: dragana@pmf.ni.ac.rs

We shall examine an additive problem of finding $(a+b)^d$ in terms of D-invertible ring elements a and b and their Drazin inverses. Also, we focus our attention to compute $\begin{bmatrix} a & c \\ b & d \end{bmatrix}^d$ when it exists. In particular, we consider the (2, 2, 0) Drazin problem, i.e. we present a formula for the Drazin inverse of $m = \begin{bmatrix} a & c \\ b & 0 \end{bmatrix}$ under suitable conditions.

Drazin inverse and generalized (pseudo) Drazin inverse in a ring

Jianlong Chen

Department of Mathematics, Southeast University
E-mail: jlchen@seu.edu.cn

The notions of the generalized Drazin inverses in Banach algebras and rings were introduced in 1996 and 2002, respectively. Because of desirable spectral property, the generalized Drazin inverse attracted widely concern. In this talk, we introduce pseudo Drazin inverse in a ring, and discuss additive and multiplicative property of (generalized, pseudo) Drazin invertibility of elements in a ring. In particular, we present Cline's formula and Jacobson's lemma for the (generalized, pseudo) Drazin inverse in a ring. We also consider Drazin invertibility of difference and product of two idempotents in a ring.

Multiplicative perturbation for the Moore-Penrose inverse and accurate solution of least squares problems

N. Castro-González

Facultad de Informática, Universidad Politécnica de Madrid, Spain
E-mail: nieves@fi.upm.es

Multiplicative perturbation theory of matrices has been widely used in the context of accurate computations of eigenvalues and singular values.
This talk will be focussed on the multiplicative perturbation theory of the Moore-Penrose inverse and its application in the context of accurate solution of least squares problems $\min_x \parallel b - Ax \parallel_2$, where $A \in \mathbb{C}^{m\times n}$ and $b \in \mathbb{C}^{m\times 1}$. The case when $A \in \mathbb{C}^{n\times n}$ is nonsingular is discussed in [F. Dopico and J. M. Molera, *Accurate solution of structured linear systems via rank−revealing decompositions*, IMA J. Numer. Anal., 32, 2012, pp. 1096-1116].
Many types of structured matrices arising in applications are very ill conditioned and standard algorithms for LS problems may compute solutions with

huge relative errors. Some examples are Vandermonde matrices which arise in polynomial data fitting and Cauchy matrices.
We show bounds for the variation of the Moore-Penrose under multiplicative perturbations, independent of the condition number $\kappa_2(A) = \| A \| \| A^\dagger \|$, and we make a comparison with the bounds in [L.-X. Cai, W.-W. Xu, and W. Li, *Additive and multiplicative perturbation bounds for the Moore − Penrose inverse*, Linear Algebra Appl., 434 (2011), pp. 480-489].
After providing multiplicative perturbation upper bounds for LS problems valid on both when A has full-rank and when A is rank-decient, we will present an algorithm to compute the minimum 2-norm solution of LS problems with high relative accuracy, i.e., with errors $\| \hat{x}_0 - x_0 \| / \|x_0\| = O(\mathsf{u})$, where $x_0 = A^\dagger b$, $\hat{x}_0$ is the computed solution and u is the unit roundoff, via an accurate rank revealing decomposition (RRD) of the coefficient matrix.
This is a joint work with
J. Ceballos, F. Dopico, J. M. Molera
Universidad Carlos III de Madrid, Spain
The research is partially supported by Project MTM2010-18057, “Ministerio de Ciencia e Innovación“ of Spain.

The additivity of the Drazin inverse for block matrices

J. Ljubisavljević
Faculty of Medicine, University of Niš, Serbia
E-mail: jelena.ljubisavljevic@medfak.ni.ac.rs

We present the new results concerning the Drazin inverse for the sum of two matrices, under conditions which are weaker than those used in some recent papers on the subject. As an application we give some new representations for the Drazin inverse of a block matrix.

Simultaneous decomposition of two EP matrices with applications

Xiaoji Liu
School of Science, Guangxi University for Nationalities
E-mail: xiaojiliu72@yahoo.com.cn

For two complex EP matrices A and B of the same order we find two "simple" matrices $A'$and B' unitarily equivalent to A and B, respectively, under a wide set of conditions. We apply this decomposition to study the non-singularity, determinants, singular values, the reverse order law for the Moore-Penrose inverse, norms, and group invertibility of certain matrices related to A and B.

Some applications of generalized inverses

Jipu Ma
Nanjing University
Tseng Yuan-Rong Functional Research Center,
Harbin Normal University
E-mail: jipuma@126.com

1. Complete Rank Theorem of Advanced Calculus

Let $f : U \subset E \mapsto F$ be a C^1-map, and $x_0 \in U$. Recall three important theorems in advanced calculus, Immersion Theorem, Submersion Theorem and Rank Theorem, the assumptions of them are that $N(f'(x_0)) = \{0\}$ and $R(f'(x_0))$ splits F, $R(f'(x_0)) = F$ and $N(f'(x_0))$ splits E and Rank $f'(x)$ is a constant near x_0, respectively, while conclusions are the same one that there exist neighborhoods U_0 at x_0 and V_0 at 0, local diffeomorphisms $\psi : U_0 \mapsto \psi(U_0)$ and $\psi : V_0 \mapsto \psi(V_0)$, such that the following equality holds:

$$f(x) = (\psi \circ f'(x_0) \circ \phi)(x), \quad \forall x \in U \qquad (*)$$

See [1] and [2]. M. Berger in [3] shows that it is not known yet whether the rank theorem in advanced calculus holds, that is, what properties of $f'(x)$ (more general than conditions given in the three theorems above) ensure equality (*) holds.

We have proved the following Rank Theorem:

Theorem (Rank Theorem): Let $f : U \subset E \mapsto F$ be a C^1-map, $x_0 \in U$ and $f'(x_0)$ be double split. Then there exist neighborhoods U_0 at x_0 and V_0 at 0, local diffeomorphisms $\psi : U_0 \mapsto \psi(U_0)$ and $\psi : V_0 \mapsto \psi(V_0)$, such that the equation (*) holds.

2. A Principle for Critical Point under Generalized Regular Constraint and Ill-Posed Lagrange Multipliers under Non-Regular Constraints

We established critical point theory under generalized regular constraint:

(1) Suppose that g from open set U in Banach space E into Banach space F is a C^1 map. Let $g'^{+}(x)$ be a generalized inverse of $g'(x), f$ be a C^1 non linear functional on U and $S =$ the preimage $g^{-1}(y_0), y_0 \in F$. If y_0 is a generalized regular value of g and $x \in U$ is a critical point of $f|_S$, then

$$f'(x) - f'(x) \circ {g'}^{+}(x) \circ g'(x) = 0 \quad \text{and} \quad g(x) = y_0,$$

for any ${g'}^{+}(x) \in GI(g'(x))$. i,e., $\lambda = f'(x) \circ {g'}^{+}(x) \in F^*$ is a Lagrange multiplier.

(2) It is proved that the above Lagrange multiplier λ is unique under regular constraint; otherwise, λ is multiplier (Ill-Posed). From this one can observe that it is very difficult to solve Euler equations with Lagrange multiplier in the case of non regular constraint.

(3) We give a principle for critical point under generalized regular constraint, no Lagrange multiplier is involved. See [13].

3. A Generalized Transversality in Global Analysis

See[14].

4. Geometrical Structure of Matrices and Operators

See[15] and [16].

5. A Geometrical Characteristic of Banach Space with C^1-Norm

References

[1] R. Abraham, J.E. Marsden, T. Ratiu, Manifold, Tensor analysis and its Applications, WPC: Springer-Verlag, (1988).

[2] E. Zeidler, Nonlinear Functional Analysis and Its Applications IV, New York-Berlin: Springer-Verlag, (1988).

[3] M. Berger, Nonlinearity and Functional Analysis, New York: Academic Press, 1976.

[4] Huang Qianlian, Ma Jipu, Perturbation analysis of generalized inverses of linear operators in Banach spaces. Linear Algebra Appl, 2004, 389: 335-364.

[5] Ma Jipu, (1.2) inverses of operators between Banach spaces and local conjugacy theorem. Chinese Annals of Math, Ser B, 1999, 20: 57-62.

[6] Ma Jipu, Rank theorem of operators between Banach spaces. Science in China, Ser A, 2000, 43: 1-5.

[7] Ma Jipu, Local conjugecy theorem, rank theorems in advanced calculus and a generalized principle for constructing Banach manifolds. Science in China, Ser A, 2000, 43: 1233-1237.

[8] Ma Jipu, A generalized preimage theorem in global analysis. Science in China, Ser A, 2001, 44: 299-303.

[9] Ma Jipu, A rank theorem of operators between Banach spaces. Front Math China, 2006, 1(1):138-143.

[10] Ma Jipu, Three classes of smooth Banach submanifolds in B(E, F). Front Math China, 2006, 1(3): 476-479.

[11] Ma Jipu, Topological and geometric property of matrix algebra. Analysis in Theory and Applications, 2007, 23: 198-200.

[12] Ma Jipu, Complete rank theorem of advanced calculus and singularities of bounded linear operators, Front. Math. China 2008, 3(2): 1-12.

[13] Ma Jipu, A Principle for Critical Point under Generalized Regular Constraint and Ill-Posed Lagrange Multipliers under Non-Regular Constraints, submitted.

[14] Ma Jipu, A Generalized Transversality in Global Analysis, Pacific J. Math., 2008, 236(2): 357-371.

[15] Ma Jipu et al., The smooth Banach submanifold $B^*(E, F)$ in B(E, F), Science in China, 2009, 52(11): 2479-2492.

[16] Ma Jipu et al., A common property of $R(E, F)$ and $B(\mathbb{R}^n, \mathbb{R}^m)$ and a new method for seeking a path to connect two operators, Science in China, 2010, 53(10): 2605-2620.

Using generalized inverses of matrices to establish closed-form solutions of LMIs

Yongge Tian

China Economics and Management Academy,

Central University of Finance and Economics
E-mail: yongge.tian@gmail.com

This talk introduces how to establish general solutions in closed-form for the three simple linear matrix inequalities (LMIs)

$$AXB \succcurlyeq C, \quad AXA^* \succcurlyeq B, \quad AX + (AX)^* \succcurlyeq B$$
$$(AXB \succ C, \quad AXA^* \succ B, \quad AX + (AX)^* \succ B)$$

in the Löwner partial ordering by using operations of the given and their generalized inverses of matrices.

Solving singular linear equations and generalized inverses

Yimin Wei

School of Mathematical Sciences, Fudan University
E-mail: ymwei@fudan.edu.cn

In this talk, we present some results on the generalized inverse and singular linear equations.

Solvability of certain quadratic operator equations and representations of Drazin inverses

Qingxiang Xu

College of Science, Shanghai Institute of Technology

Department of Mathematics, Shanghai Normal University
E-mail: qxxu67@sit.edu.cn

Let E be a Banach space, $B(E)$ be the set of bounded linear operators on E, and I be the identity operator on E. In this talk the solvability of the quadratic operator equations

$$AX + X^2 = B \text{ and } XA + I = XBX$$

is studied, where $A \in B(E)$ is invertible and $B \in B(E)$ is nilpotent. It is proved that the first quadratic operator equation is solvable if furthermore $AB = BA$ or $E = C^n$ for $n \in \{2, 3\}$ and $B^2 = 0$. A counterexample is given such that the second quadratic operator equation is unsolvable. As an application, in the general setting of bounded linear operators on Banach spaces, a new expression of the Drazin inverse $\begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^D$ is given under the condition that $AB \in B(E)$ are Drazin invertible and $AB = BA$.

The multiplicative perturbation bounds of the group inverse and oblique projection

Lingsheng Meng, Bing Zheng

School of Mathematics and Statistics, Lanzhou University
E-mail: bzheng@lzu.edu.cn

In this paper, the multiplicative perturbation bounds of the group inverse and related oblique projection under general unitarily invariant norm are presented by using the decompositions of $B^\sharp - A^\sharp$ and $BB^\sharp - AA^\sharp$.