State University of Novi Pazar

BOOK OF ABSTRACTS
CPMMI 2020
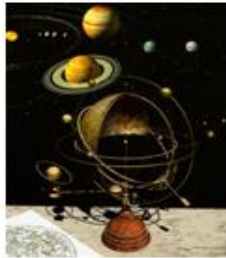
Novi Pazar, 21–22 September 2020.

BOOK OF ABSTRACTS

6th International Conference CONTEMPORARY PROBLEMS OF MATHEMATICS, MECHANICS AND INFORMATICS

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KEYNOTE LECTURE
CPMMI 2020

ON EXTENSIONS OF MAPPINGS

Anatoly B. Antonevich[1], Ćemal Dolićanin[2]
[1]Belarusian State University, Belarus
[2]State University of Novi Pazar, Serbia

The general concept of an extension is introduced for mapping $f: X_0 \to Y_0 \subset Y$, where X and Y are topological spaces and $X_0 \subset X$, $Y_0 \subset Y$. Three constructions of such extensions are proposed and corresponding examples are given, and in one of them, a transition to the extension coincides with the Gelfand transform. The peculiarity of these constructions is that the domain of definition of the extended transformation does not belong to X, but it is a bundle over a subset of X.

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PLENARY SESSION
CPMMI 2020

HOPF BIFURCATIONS IN SOME BIOCHEMICAL MODELS

Valery Romanovski

Center for Applied Mathematics and Theoretical Physics, University of Maribor, Slovenia

We propose an approach based on the elimination theory of computational algebra to find conditions for existence of Hopf bifurcations in polynomial systems of ordinary differential equations. Some applications to the investigation of biochemical models are discussed.

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SOME NEW RESULTS IN THE THEORY OF PSEUDO-DIFFERENTIAL OPERATORS

Stevan Pilipović

Faculty of Sciences, University of Novi Sad, Serbia

This paper presents some of the recent results of pseudo-differential operators. After the introduction related to the Weyl-Hormader theory, it is shown the equivalence of ellipticity and for Fredholmness for a wide class of pseudo-differential operators satisfying with a natural assumption. An interesting example is given, as well.

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ANGLES AND PERTURBATIONS OF COMPLEMENTED SUBSPACES OF A HILBERT SPACE

Dragan Đorđević

Faculty of Sciences and Mathematics, University of Niš, Serbia

We investigate angles between subspaces of a Hilbert space, and also prove a result concerning the perturbation of a complemented subspace.

***

ATOMIC DECOMPOSITIONS IN WEIGHTED BERGMAN SPACES OF ANALYTIC FUNCTIONS ON STRICTLY PSEUDOCONVEX DOMAINS

Miloš Arsenović
Faculty of Mathematics, University of Belgrade, Serbia

Several maximal function type operators acting on analytic functions defined on a smoothly bounded strictly pseudoconvex domain in $\mathbb{C}^n$ are investigated, boundedness of such oparators in certain norms are proved. These results are then applied to obtain atomic decomposition of weighted Bergman spaces on such domains. The atoms used are real valued atoms.

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ON THE THERMODYNAMICAL RESTRICTIONS IN ISOTHERMAL DEFORMATIONS OF FRACTIONAL BURGERS MODEL

Teodor M. Atanacković

Faculty of Technical Sciences, Institute of Mechanics, University of Novi Sad, Serbia

We investigate restrictions on a constitutive equation for a fractional Burgers model of viscoelastic fluid that follow from the weak Entropy inequality under isothermal conditions. Our results extend the ones obtained by Bagley–Torvik. Moreover, we give restrictions for the more general types of constitutive equations. Examples of Section 3 show preferences of our approach.

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ON SOME RESULTS OF SCIENTIFIC AND HIGHER EDUCATION COOPERATION WITH PROF. ĆEMAL DOLIĆANIN

Neda Bokan
University of Belgrade, Serbia

Prof. dr Neda Bokan spoke about her long friendship with prof. dr. Ćemal Dolićanin, especially from the Master Studies of Faculty of Mathematics, University of Belgrade, until today, and beyond. A special review was made of their joint books *Differential Geometry, Problems and Solutions* and *Selected Chapters from Mathematics*, which were published in 2016 and 2018, respectively. These books are results of their cooperation and experience in higher education during these decades.

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ON SUMMATION OF p-ADIC SERIES

Branko Dragović
Institute of Physics, University of Belgrade,
Mathematical Institute of SASA, Serbia

Summation of the p-adic functional series $\sum \varepsilon^n n! \, P_k^\varepsilon(n; x) x^n$, where $P_k^\varepsilon(n; x)$ is a polynomial in x and n with rational coefficients, and $\varepsilon = \pm 1$, is considered. The series is convergent in the domain $|x|_p \leq 1$ for all primes p. It is found the general form of polynomials $P_k^\varepsilon(n; x)$ which provide rational sums when $x \in \mathbb{Z}$. A class of generating polynomials $A_k^\varepsilon(n; x)$ plays a central role in the summation procedure. These generating polynomials are related to many sequences of integers. Details can be found in my review paper: Contemporary Mathematics 704 (2018) 127–138; arXiv:1702.02569 [math.NT].

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MY COOPERATION AND FRIENDSHIP WITH PROF. ĆEMAL DOLIĆANIN

Dragić Banković
University of Kragujevac, Serbia

Prof. dr Dragić Banković spoke about the long-term cooperation with prof. Ćemal Dolićanin, and especially about the preparation of elaborations for the accreditation of study programs of Basic and Master Academic Studies of Mathematics and Informatics-Mathematics.

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SESSION 1
CPMMI 2020

AN OVERVIEW ON BITCOIN AND BLOCKCHAIN TECHNOLOGY

Miodrag Mihaljević
Mathematical Institute of SASA, Serbia

The first part of the talk addresses Bitcoin, a widespread digital currency based on cryptography and other elements of blockchain technology. Bitcoin has introduced new applications of cryptography and it is the first "big application" of blockchain technology. The second part of the talk addresses the main issues of blockchain technology which appears as a technology for a lot of applications.

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SELECTIVE PROPERTIES OF FUZZY 2-METRIC SPACES

Ljubiša Kočinac[1], Vildan Cetkin[2], Diana Dolićanin Đekić[3]

[1]*University of Niš, Serbia*
[2]*Kocaeli University, Turkey*
[3]*State University of Novi Pazar, Serbia*

We study some topological properties of fuzzy 2-metric spaces related to the classical covering properties of Menger, Hurewicz and Rothberger. Recall that the notion of 2-metric space was introduced by Gahler in the 1960s. Fuzzy 2-metric spaces are defined with the help of the well-known continuous triangular norm (t-norm). Our study is concentrated on boundedness properties of fuzzy 2-metric spaces. A fuzzy 2-metric space X is said to be F-2-*Mengerbounded* (shortly FM_2-bounded) if for each sequence $(\varepsilon_n)_{n\in\mathbb{N}}$ of elements of $(0,1)$ and each $t > 0$ there is a sequence $(A_n)_{n\in\mathbb{N}}$ of finite subsets of X such that $X = \bigcup_{n\in\mathbb{N}}\bigcup_{a\in A_n} B(a,\varepsilon_n,t)$, where $B(a,\varepsilon_n,t)$ is an open ball. We discuss also infinitely long two-person games naturally associated to these boundedness properties.

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SOME NOTES TO THE CLASS OF HQR AND HQC MAPPINGS

Miljan Knežević

Faculty of Mathematics, University of Belgrade, Serbia

We give a new idea how to consider some important properties of the hyperbolic metric to obtain results of the Ahlfors-Schwarz-Pick type for the class of HQR (harmonic and quasiregular) and HQC (harmonic and quasiconformal) mappings. By using some important estimates for hyperbolic partial derivatives which we obtained, we discus the hyperbolic bi-Lipschicity of those mappings.

***

AN INTRODUCTION TO LOGICS WITH PROBABILITY OPERATORS

Zoran Ognjanović
Mathematical Institute of SASA, Serbia

The problems of representing, and working with, uncertain knowledge are ancient problems dating, at least, from Leibnitz. In the last decades there is agrowing interest in the field connected with applications to computer science and artificial intelligence. Researchers from those areas have studied uncertain reasoning using different methods. Some of the proposed formalisms for handling uncertain knowledge are based on logics with probability operators. The aim of this presentation is to provide an introduction to such formal systems and results presented in the papers listed below. The main focus is related to mathematical techniques for infinitary probability logics used to obtain results about proof-theoretical and model-theoretical issues: axiomatizations, completeness, compactness, decidability.

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SUMMARY OF THE BOOK
DIFFERENTIAL GEOMETRY AND ITS VISUALIZATION

Ćemal Dolićanin[1], Eberhard Malkowsky[2], Vesna Veličković[3]

[1]State University of Novi Pazar, Serbia
[2]Department of Mathematics, University of Giessen, Germany
[3]Faculty of Sciences and Mathematics, University of Niš, Serbia

Differential geometry is a very powerful tool for exploring curves and surfaces. It provides many useful formulas for analyzing their shapes and properties. It is even more useful to see graphics that visualize appropriate concepts or watch animations that demonstrate them. In this talk, we give an overview of some of the concepts of Differential Geometry and illustrate them with visualizations and animations.

In view of professor Dolićanin 75th birthday, we want to emphasise that his research area is Differential Geometry. In this talk we give a summary of the joint book *Differential Geometry and its Visualization*, by Dolićanin, Malkowsky and Veličković, that is about to be printed.

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SESSION 2
CPMMI 2020

MATHEMATICS AND PHYSICS IN WOS: SERBIA AND THE STATE UNIVERSITY OF NOVI PAZAR, 2015–2019

Đuro Kutlača, Lazar Živković
University of Belgrade, Mihajlo Pupin Institute – Science and Technology Policy Centre, Serbia

This paper analyses the publication of scientific papers in mathematics and physics in journals registered by WoS authors from Serbia and, in particular, authors who are associates of the State University of Novi Pazar in the period 2015–2019. In addition, the connection of associates of the State University of Novi Pazar with other universities and institutions in Serbia in co-authorship of articles in the field of mathematics in the same period is analysed.

The aim of the analyses is to research the intensity of publishing in these two fields of science and, in particular, the intensity of networking of associates of the State University of Novi Pazar with other universities and institutions in Serbia.

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MUSIC SCALE TEMPERATION: ORIGIN OF THE IDEA, ADVANTAGES AND DISADVANTAGES OF VARIOUS PRACTICES

Svetlana Stojanović Kutlača[1], Sonja Marinković[2]
[1]Music school "Josip Slavenski", Serbia
[2]Faculty of Music, University of Arts in Belgrade, Serbia

The paper sheds light on some aspects of the connection between music and mathematics concerning the tuning of instruments and the temperation system. The authors discuss questions about the origin of the idea of temperation and point out the advantages and disadvantages of various modern practices which are especially important in the contemporary digital era.

The connection between music and mathematics, based in ancient times, develops along with the development of musical instruments and artistic music, and its importance is intensified in the modern age of digitalization, characterized by, in addition to the need to innovate production and improve the quality of musical reproduction, the desire to preserve the authenticity of sound and musical interpretations.

The connection between music and mathematics rests on Pythagoras' research into the principles of the origin of tone and the formation of natural tone scales – modus, necessary for the tuning of lyre as multi-string instrument. Plato extends Pythagoras' theory by considering the individual characteristics of modus according to the peoples who use them and the possibilities of their coexistence in the wider community – the "Ideal State".

Ancient ideas about music, adopted in the period of medieval Christianity, are modified in the Renaissance period for practical reasons – tuning, new musical instruments, organ and harpsichord, developed from old ones, to which the keyboard mechanism is attached, a clear indicator of the adopted 12-tone system. Temperament – a twelve-tone scale tuning – is based on an approximation, the ability of the hearing to accept and assimilate small imperfections in tones and intervals. Different types of Baroque and Renaissance temperaments are based on knowledge of Pythagorean theory and awareness of the different character – the "energie of modus". The mathematical possibility of equal temperament was discovered with the discovery of the 12th root from number 2, as early as the 16th century, but until the 19th century

musicians avoided it because it caused a total unification of sound colors and spectrum of sound material.

In the mid-18th century, the existence of sound colors was explored by Jean Flip Ramo, a scientist, encyclopedist, creator of the science of musical harmony and a significant composer. Long before the time of electronics, Ramo performed numerous experiments with organs, spectral analysis of the musical color of instruments and the acoustics of rooms. During the 19th century, due to the need to chromatize symphony orchestral instruments, the equal temperament of classical instruments was adopted as it is today. During the subsequent musical epochs, other ways of developing the musical language were adopted, and the concept of harmony changed. Awareness of authentic sounding, as a necessary aspect of interpretation, today influences the reproduction of musical works through the choice of appropriate instruments, spaces and specific interpretative procedures.

Digitization characterizes the modern age and penetrates all domains of culture, including the musical space of the construction of musical instruments, interpretation and reproduction of sound. It is

precisely because of the development of digitalization that the possibility of re-examining the quality and meaning of music and musical works opens up again. Music is a language, as well as in the domain of the language, sometimes we are giving preference to the wide and clear availability of information, but it must not be neglected or limited the possibility of preserving the wide range of qualities and individual finesse that artistic speech contains.

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APPROXIMATING FIXED POINTS FOR GENERALIZED NON-EXPANSIVE MAPPINGS WITH APPLICATIONS

Javid Ali

Department of Mathematics, Aligarh Muslim University, India

In this talk, we show that the classes of generalized non-expansive mappings due to Hardy and Rogers and the mappings satisfying Suzuki's condition (*C*) are independent and study some basic properties of generalized non-expansive mappings. Further, we introduce a new iterative scheme, called JF iterative scheme, and prove convergence results for generalized non-expansive mappings due to Hardy and Rogers in uniformly convex Banach spaces. Moreover, we show numerically that JF iterative scheme converges to a fixed point of generalized non-expansive mappings faster than some known and leading iterative schemes. As an application, we utilize newly defined iterative scheme to approximate the solution of a delay differential equation. Also, we present some nontrivial illustrative numerical examples to support main results.

Finally, we also approximate common fixed points of the generalized non-expansive mapping via one step iterative scheme in

uniformly convex Banach space. We utilize the result to solve image recovery problem in Banach space. Some examples are furnished in the support of the results.

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ON SPACE DEBRIS DEORBITING AND A NEW APPROACH TO THE PROBLEM OF ATTITUDE STABILIZATION OF ELECTRODYNAMIC SPACE TETHER SYSTEM

Alexey A. Tikhonov
Saint Petersburg State University, Russia

The paper deals with the problem of space debris deorbiting. Nowadays the promising approach to the problem is associated with the use of the drag effect of Ampere forces acting upon electrodynamic space tether system (EDTS) in geomagnetic field. EDTS with conducting tether carrying electric current and moving along near-Earth circular orbit is under consideration. The drag effect of Ampere forces is most intensive if the tether is oriented along the local vertical in near-Earth space. In the case of the ordinary heavy tether this orientation is stable in the central Newtonian gravitational field. At the same time, it is established that under the action of the Ampere torque the vertical orientation of the electrodynamic tether is destroyed. The problem of EDTS instability is known. A number of works have been devoted to solving this problem. Among the possible approaches to its solution, it is proposed to switch off the current

periodically or to use control of the current strength flowing along the tether.

In the present paper, an original design scheme for constructing an EDTS has been proposed in order to solve the EDTS instability problem. This scheme includes a negatively charged collector at the lower end of the tether and a positively charged collector at the upper end of the tether. The magnitude of the charge on the negatively charged collector is controlled by electronic emitters. Analytical studies and computer simulations have shown that the Lorentz torque acting on the EDTS due to charged collectors at the ends of the tether significantly expands the stability area for the vertical position of the tether. Controlling the charge on a negatively charged collector in accordance with the current attitude motion of the tether makes it possible to create a control component of Lorentz torque with a dissipative-like nature. The simultaneous operation of the restoring and dissipative-like components of the Lorentz torque makes it possible to ensure the asymptotic stability of the vertical position of the tether without the need to turn off the electric current flowing along the tether. The complexity of EDTS construction is not significant, since it does not imply the introduction of fundamentally

new devices in its structure in comparison with those normally used in EDTS. The proposed device and control method can be used to stabilize the space tether system in near-Earth space in order to increase the efficiency of its operation in the process of space debris deorbiting.

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SIMPLER PROOFS OF SOME RECENT RESULTS OF F-SUZUKI CONTRACTION IN b-METRIC SPACES

Ersin Gilić[1], Diana Dolićanin Đekić[2,1], Zoran D. Mitrović[3], Dženis Pučić[1], Hassen Aydi[4,5]

[1]*State University of Novi Pazar, Serbia*
[2]*Faculty of Technical Sciences in Kosovska Mitrovica, University of Priština, Serbia*
[3]*Faculty of Mathematics and Statistics, Ton Duc Thang University, Vietnam*
[4]*Université de Sousse, Institut Supérieur d' Informatiqueet des Techniques de Communication, Tunisia*
[5] *China Medical University Hospital, China Medical University, Taiwan*

Much simpler and shorter proofs of some recent results in the context of generalized F-Suzuki-contraction mappings in b-complete b-metric spaces is provided in this paper. Our results generalized, complement and improve many known results in the existing literature using the new approach for the proof that a Picard sequence is b-Cauchy. Further, some new contractive conditions are provided here to illustrate the usability of the obtained theoretical results.

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ON THE INFLUENCE OF NEAR EARTH PLASMA ON FUNCTIONING OF ELECTRODYNAMIC SPACE TETHER SYSTEM FOR SPACE DEBRIS DEORBITING

Andrey B. Yakovlev, Alexey A. Tikhonov
Saint Petersburg State University, Russia

It is known that more than semi-centennial activity in near Earth space, along with the successful solution of a wide range of fundamental and applied problems, has led to some negative consequences. Escalating pollution of a near-Earth space by products of manmade activity (so-called «space debris» (SD)) is the most serious of these problems. Large SD objects with sizes from some centimeters to dozens of meters are especially dangerous for orbital spacecrafts since the collision with such SD object may lead to catastrophic consequences. One of perspective methods for mitigation of large-scale SD component is based on the usage of electrodynamic space tether systems (EDTS) for SD deorbiting. In this paper we analyze various models of interaction between EDTS and surrounding near-Earth plasma. The influence of changing space plasma parameters on EDTS efficiency is investigated.

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SOFTWARE ENGINEERING:
A FORTY YEAR RESEARCH RETROSPECTIVE

Ejub Kajan
State University of Novi Pazar, Serbia

This paper presents author experience with software development during forty years. This period is divided into three era: closed and open systems and Internet of Everything, each covered by challenges, issues and solutions. The main stream is given from the interoperability perspective and avored by security issues. Variety of applications domains, that author faced during his carrier, each with own challenges, is presented. Example include computer graphics systems, special-purpose solutions, business-to-business, business process management, e-government, and smart cities.

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ON THE DOMAIN OF FOUR-DIMENSIONAL SEQUENTIAL BAND MATRIX IN SOME DOUBLE SEQUENCE SPACES

Orhan Tug

Department of Mathematics Education, Tishk International University, Iraq

The concept of four-dimensional generalized difference matrix $B(r, s, t, u)$ and its domain on some double sequence spaces was recently introduced and studied by Tug and Basar. In this present paper, we define four-dimensional sequential band matrix $B'(r', s', t', u')$ and introduce some new double sequence spaces $B'(M_u)$, $B'(C_p)$, $B'(C_{bp})$, $B'(C_r)$ and $B'(L_q)$ where $0 < q < \infty$ whose $B'(r', s', t', u')$-transforms are in the double sequence spaces M_u, C_p, C_{bp}, C_r and L_q, respectively. Firstly, we show the isomorphism between the new double sequence spaces $B'(M_u)$, $B'(C_{(v)})$, $B'(L_q)$ and classical double sequence spaces $M_u, C_{(v)}, L_q$, respectively. Then, we prove some inclusion relations under some strict conditions and we determine dual spaces of these new double sequence spaces. Moreover, we characterize some new classes of four-dimensional matrix mappings related to our new double sequence spaces. Finally, we conclude the paper with some significant consequences.

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ON THE NORM OF IDEMPOTENT OPERATORS IN A HILBERT SPACE

Vladimir Rakočević
University of Niš, Serbia

Let R and K be subspaces of a Hilbert space H, and let P_R and P_K denote the orthogonal projections of H onto these subspaces. Buckholtz has proved that the operator $P_R - P_k$ is invertible if and only if H is the direct sum of R and K. In this case there exists a linear idempotent M with range R and kernel K, and $\parallel P_K P_R \parallel < 1$. In this note we give a precise value of $\parallel M \parallel$, get a sharper estimate of a result of Vidav, and prove that $\parallel P_K P_R \parallel$ is equal to the gap between R and $K^\perp$.

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SESSION 3
CPMMI 2020

NETHER'S THEOREM FOR VARIATIONAL PROBLEMS OF HERGLOTZ TYPE WITH REAL AND COMPLEX ORDER FRACTIONAL DERIVATIVES

Marko Janev
Mathematical Institute of SASA, Serbia

A variational principle of Herglotz type for Lagrangian that depend on fractional derivatives of both real and complex order is formulated and the invariance of this principle is studied. Necessary and sufficient conditions for an infinitesimal transformation group to be a symmetry group are obtained. These conditions extend the classical results, valid for integer order derivatives. A generalization of Nether's theorem leading to conservation laws for Herglotz fractional Euler Lagrange equation proved. An approximation of a fractional Euler–Lagrangian equation by a system of integer order equations is used to formulate an approximated invariance condition and corresponding conservation laws. Results are illustrated by several concrete examples.

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ON THE MINIMUM SECOND ZAGREB INDEX OF TREES WITH SMALL PARAMETERS

Bojana Borovićanin[1], Emir Zogić[2]

[1]Department of Mathematics and Informatics, Faculty of Science, University of Kragujevac, Serbia
[2]Department of Mathematics, State university of Novi Pazar, Serbia

Let G be a simple graph with vertex set $V = V(G) = \{v_1, v_2, \dots, v_n\}$ and edge set $E = E(G)$. Denote by $d_i = d_i(G)$ the degree (number of neighbors) of the vertex $v_i \in V(G)$. The second Zagreb index $M_2(G)$ of a graph G is defined as

$$M_2(G) = \sum_{v_i v_j \in E(G)} d_i d_j.$$

The second Zagreb index is one of the oldest vertex-degree-based molecular structure descriptors, introduced in the 1970s. Recently, there has been great interest in studying extremal graphs that minimize (or maximize) second Zagreb index in different classes of graph. Lower bounds on the second Zagreb index of trees with given small parameters such as diameter, matching number and domination number are determined and the extremal trees are characterized, as well.

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SOME REMARKS ON KIRCHHOFF INDEX

Emir Zogić[1], Edin Glogić[1], Emina Milovanović[2], Igor Milovanović[2]

[1]Department of Mathematics, State University of Novi Pazar, Serbia
[2]Faculty of Electronic Engineering, University of Niš, Serbia

Let $G = (V, E)$, $V = \{v_1, v_2, \dots, v_n\}$, be a simple connected (n, m)-graph with a sequence of vertex degrees $d_1 \geq d_2 \geq \cdots \geq d_n > 0$, $d_i = d(v_i)$. Denote with $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$, the Laplacian eigenvalues of G. The Kirchhoff index of a graph G is defined

$$K f(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i}.$$

Further, denote by S_α a sum of powers of the Laplacian eigenvalues of G, that is

$$S_\alpha = \sum_{i=1}^{n-1} \frac{1}{\mu_i^\alpha},$$

where α is an arbitrary real number.

The zeroth order Randić index is a vertex-degree-based topological index defined as

$$^{0}R_{\alpha} = \sum_{i=1}^{n} d_{i}^{\alpha},$$

where α is an arbitrary real number.

In this paper we will prove the following two general inequalities:

$$Kf(G) \geq \frac{nS_{\alpha}(G)^2}{S_{2\alpha+1}(G)},$$

and

$$Kf(G) \geq \frac{(n-1)\,^{0}R_{\alpha}(G)^2 - \,^{0}R_{2\alpha+1}(G)}{^{0}R_{2\alpha+1}(G)}.$$

By the appropriate choice of parameter α we obtain a number of new and old bounds for the Kirchhoff index.

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ON THE MODIFIED FIRST ZAGREB INDEX AND COINDEX

Marjan Matejić, Igor Milovanović, Emina Milovanović
Faculty of Electronic Engineering, University of Niš, Serbia

Let $G = (V, E)$, $V = \{v_1, v_2, \ldots, v_n\}$, be a simple (n, m)-graph without isolated vertices, and with a sequence of vertex degrees $\Delta = d_1 \geq d_2 \geq \cdots \geq d_n = \delta > 0$, $d_i = d(v_i)$. If vertices v_i and v_j are adjacent in G, we write $i \sim j$, otherwise $i \nsim j$.

The modified first Zagreb index, $^m M_1(G)$, and its coindex $\overline{^m M_1}(G)$ are defined as

$$^m M_1(G) = \sum_{i \sim j} \left(\frac{1}{d_i^3} + \frac{1}{d_j^3} \right) = \sum_{i=1}^{n} \frac{1}{d_i^2},$$

and

$$\overline{^m M_1}(G) = \sum_{i \nsim j} \left(\frac{1}{d_i^3} + \frac{1}{d_j^3} \right) = \sum_{i=1}^{n} \frac{n - 1 - d_i}{d_i^3}.$$

In this paper we prove a number of inequalities that set up upper and lower bounds for ${}^mM_1(G)$ and $\overline{{}^mM_1}(G)$. Particulary, we consider the cases when G is a tree, $G \cong T$, and unicyclic graph, $G \cong U$.

Also we prove the inequality

$$ {}^mM_1(G) + \overline{{}^mM_1}(G) \geq \frac{n^4(n-1)}{2m^3}, $$

which reveals a relationship between the modified first Zagreb index and its coindex.

***

KEPLER'S EQUATION AND ANGULAR MOMENTUM: HISTORICAL PERSPECTIVE, CRITICAL ANALYSIS AND IMPLICATIONS FOR DEVELOPMENT OF MECHANICS, MATHEMATICS AND PHYSICS

Slobodan Nedić
Faculty of Technical Sciences, University of Novi Sad, Serbia

After 410 years of the formulation and publication (in *Astronomia Nova*, 1609) of the Kepler's Equation, which relates the eccentric (and, intermediately, the true) anomaly of the planetary trajectories to the uniformly flowing time, in accordance with his Second (surface) law, in this paper are perceived with it connected certain deficiencies of the orbital mechanics and dynamics, caused by absence of the Kepler's accompanying physical considerations, and which are repercuted in: reliance on the so-called Invariants – the First integrals of Energy and of the Angular Momentum, implicit Conservativeness, canonic formalism and omnipresence of the Symmetry principle, as well as the essential lacking of the explicit centrifugal force and its substitution by the fictitious one. It is given a survey of the Kepler's strivings and the results attained, as well as of his key role in the historical development of the mechanics, physics,

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astronomy and astrophysics, and the science in general – through insights in the branching of the science development over Newton (instead over Descartes and Leibniz), incomplete congruence among his physical considerations and the ultimately formulated laws, as well as the paradoxality reflected in the lack of fulfillment of the first Energy integral, and also of the non-existent transverse acceleration 'implied' by the Kepler's Second law. In support of the justification of the neglected development direction and the fundamentality of the Kepler's insights in the need for both the attractive an repulsive interactions of the orbital and central body – the Sun, the Kepler–Ermakov second order non-linear differential equation has been reaffirmed along its adequacy for the phenomenological modeling of dynamic interactions on all the 'scales' in Nature: with brief reference to the "General Aetherodynamics" of V.A. Atsukovsky, as the basis for reuse and justification of insights/results of Descartes, Leibniz, Boscovich, D'Alambert, Engels, H. Strache, M. Petrović, M. Milanković, and P. Savić. Implications to elliptic integration and connection between physical and mathematical continua will be mentioned.

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C-CLASS AND PAIR UPPER CLASS FUNCTIONS AND OTHER KIND OF CONTRACTIONS IN FIXED POINT THEORY

Arslan Ansari[1], Anita Tomar[2]

[1]*Department of Mathematics, Islamic Azad University, Iran*
[2]*Department of Mathematics, Government Degree College Thatyur, India*

In 2014 was introduced C-class and pair upper Class functions that cover more papers before and after that, base on them some other ideas like: 1-1-upclass functions, multiplicative C-class functions, inverse-C-class functions, CF-simulation functions was planed. In this glance we look for some condition that can use them or can not.

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REGULARITY VS. COMPACTNESS

Eberhard Malkowsky
Department of Mathematics, University of Giessen, Germany

A well-known result by Cohen and Dunford characterises the class of all regular compact linear operators. Consequently, a regular matrix transformation cannot be compact. This means that if c denotes the set of all complex sequences $x = (x_k)_{k=1}^{\infty}$ of complex numbers, then an infinite matrix that maps c into c and preserves the limits cannot be compact. We outline our new proof of this result which applies the theory of BK spaces from functional analysis and summability, and uses the Hausdorff measure of noncompactness. Furthermore, we present similar recent results involving the spaces

$$w_p = \left\{ x = (x_k)_{k=1}^{\infty} : \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} |x_k - \xi|^p = 0 \text{ for some } \xi \in \mathbb{C} \right\}$$

and

$$[c] = \left\{ x = (x_k)_{k=1}^{\infty} : \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} |kx_k - (k-1)x_{k-1} - \xi| \text{ for some } \xi \in \mathbb{C} \right\}$$

of sequences that are strongly summable by the Cesaro method of order 1 with index $p \in [1, \infty)$, and strongly convergent, respectively.

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UNIFIED EXPLICIT FORMULA FOR GENERALIZED STIRLING NUMBERS

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Let $\{p_n(x),\ n = 0,1,\dots\}$ be a class of polynomials, and consider the double sequence $S(n,k),\ n = 0,1,\dots,\ k = 0,1,\dots$ obtained by expanding the polynomials $p_n(x)$ in a series of factorials, namely

$$p_n(x) = \sum_{k=0}^{n} S(n,k)x^{(k)}.$$

In this paper we discuss some general results relating to the generalized numbers $S(n,k)$.

Some special cases:

1. If $p_n(x) = x^n$, we obtain the usual Stirling numbers of the second kind.

2. If $p_n(x) = (x-a)^n$, we are led to the non-central Stirling numbers of the second kind.

3. If $p_n(x) = (-x)^n$, or more generally $p_n(x) = (sx)^n$, we obtain the Lah or C-numbers.

4. If $p_n(x) = (sx + r)^n$, the relating numbers are the Gould–Hopper numbers.

5. If $p_n(x) = (x - a_0)(x - a_1)\cdots(x - a_{n-1})$, we have generalized non-central Stirling numbers studied recently by G. V. Milovanović and N. P. Cakić.

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SESSION 4
CPMMI 2020

EXISTENCE OF SOLUTIONS OF NONLINEAR AND NON-LOCAL FRACTIONAL BOUNDARY VALUE PROBLEMS

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In this paper, we establish new results for non-local boundary value problems. In particular, we consider the following fractional analog of the thermostat model:

$$^{C}D^{\alpha}u(t) + f\big(t, u(t)\big) = 0, \quad 0 < t < 1,$$

subject to the boundary conditions

$$u'(0) = -\delta[u], \quad u'(1) = \beta[u] - \frac{1}{b}u(\eta), \quad b > 0, \quad \eta \in [0,1].$$

Here $1 < \alpha < 2$, $^{C}D^{\alpha}$ denotes the Caputo fractional derivative of order α, $\delta[\cdot]$, $\beta[\cdot]$ are suitable linear and continuous functionals on $C[0,1]$ and $f : [0,1] \times R \to [0, +\infty)$ is a continuous function. Since the associated integral equation has a kernel that is not bounded above and changes its sign, so that, the positive sign of the possible solutions is not ensured.

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ALTERATIONS OF EYE TISSUES DUE TO AGING AND DISEASE IN TERMS OF REAL AND COMPLEX-CONJUGATE DERIVATIVES

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We analyze changes of viscoelastic properties due to myopia and glaucoma, as well as changes of lenses due to aging. In doing so the concept of a change was taken in terms of fractional derivatives of real and complex-conjugate order.

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CONTROL CHARTS BASED ON QUANTILES – NEW APPROACHES

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Contemporary development of statistical quality control includes researches on different control charts, which could be easily implemented in production processes due to facilities offered by computers. New control charts give more information about production processes than the conventional ones, that's why there is a lot of investigation in this area of applied statistics. In this paper we shall explain some new ideas concerning construction of control charts based on quantiles, empirical distribution function and p-value.

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NEW SHAPE FUNCTION IN THE FREE-VIBRATION ANALYSIS OF ANTISYMMETRIC CROSSPLY COMPOSITE LAMINATES

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The paper analyzes the problem of free vibrations in antisymmetric crossply laminates. A new shape function which is used in higher order shear deformation theories has been introduced. The comparative analysis was performed with the known shape functions. The procedure for obtaining dynamic equations of motion in the Matlab software package has been developed. For theoretical considerations, the module with the symbolic variable has been used. The paper shows which of the existing shape functions are applicable in the free vibration analysis of antisymmetric crossply laminates. The advantages and disadvantages of the newly developed shape function are clearly highlighted. Analytical procedures have been used to obtain the results of partial differential equations, based on Navier's solutions. Numerical integration procedures were used as an integral

part of the developed Matlab codes for those shape functions where it was necessary. The results are presented in a table and figures. The procedure itself has been verified by comparison with the reference results from the literature.

***

NEW HYBRID CONJUGATE GRADIENT METHOD AS A CONVEX COMBINATION OF HESTENES–STIEFEL AND DIXON CONJUGATE GRADIENT METHODS

Snežana Đorđević

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In this work we give one new hybrid conjugate gradient method. This new method can be seen as a convex combination of two well-known conjugate gradient methods. We prove that the presented method satisfies the sufficient descent condition. We also prove its global convergence under strong Wolfe conditions. Numerical experiments show that this new method is the efficient one.

***

RANGE DISPARITY EVALUATION BY GRADIENT SIMILARITY

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Image quality assessment has been the subject of a lot of researches in the last years. However, little progress has been made in developing quality assessment measures for range images evaluation, where the percentage of bad matched pixels (BMP) is the commonly used measure to evaluate the performance of stereo algorithms. The BMP requires a disparity error tolerance and ignores the relationship between range and disparity. To find other alternatives to this measure and to overcome these inconveniences, in this paper, a gradient-based quality metric for range disparity maps evaluation is presented. The metric operates in a manner similar to original implementation, but allows special handling of missing data and ignores the overwhelmingly nonlinear nature of the human visual system. The utility of this metric is demonstrated by evaluating a set of two-frame stereo correspondence algorithms used in the well-known Middlebury

Stereo Vision dataset. The new metric possesses features as an alternative choice for assessing the quality of range disparity maps.

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HOMOMORPHIC CRYPTOGRAPHY

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Encrypting data in a conventional way with a public key and decrypting it with a secret key brings privacy issues especially on the clouds. Holding the key, the service providers have exclusive rights on the data. Homomorphic encryption addresses this issue since it allows any third party to operate on the encrypted data without decrypting it.

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ON SOME BERNSTEIN OPERATORS

Faruk Özger

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Bernstein polynomials are a powerful tool for replacing a lot of arduous calculations carried out for continuous functions with friendly calculations on approximating polynomials. For this reason, many researchers are interested to work on Bernstein operators with a view of studying end-points interpolation, convergence, shape preserving properties and many others. In this study, we focus on some Bernstein operators and their approximation properties.

***

THE OPTIMAL PERTURBATION BOUNDS FOR THE CORE INVERSE IN C^* ALGEBRAS

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The closed form and optimal perturbation bounds of the core inverse under some conditions are already investigated for the matrix case. In this paper these bounds are extensively studied in the context of arbitrary C^*-algebra. We present optimal perturbation bounds of the core inverse in C^*-algebras under two-sided and one-sided conditions. Obtained expressions for the perturbation of the core inverse are simple and perturbation bounds are sharp.

***

DETECTING NETWORK ANOMALIES AND INTRUSIONS

Ljiljana Trajković

Simo Fraser University, Canada

The Internet, social networks, power grids, gene regulatory networks, neuronal systems, food webs, social systems, and networks emanating from augmented and virtual reality platforms are all examples of complex networks.Collection and analysis of data from these networks is essential for their understanding. Traffic traces collected from various deployed communication networks and the Internet have been usedto characterizeand model network traffic, analyze network topologies, and classify network anomalies. Data mining and statistical analysis of network data have been employed to determine traffic loads, analyze patterns of users' behavior, and predict future network traffic while spectral graph theory has been applied to analyze network topologies and capture historical trends in their development.Machine learning techniques have proved valuable for predicting anomalous traffic behavior and for classifying anomalies and intrusions in communication networks. Applications of these tools

help understand the underlying mechanisms that affect behavior, performance, and security of computer networks.

***

WEB BASED AUGMENTED REALITY – CASE STUDY IN E-LEARNING SYSTEM BASED ON INTERACTIVE LEARNING OF PROGRAMMING

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Augmented Reality is an emerging technology, where the perception of the user is enhancedby the seamless blending between a realisticenvironment and computer-generated virtual objectscoexisting in the same space. This allow placement of additional informations (images, videos, 3D models, sounds, etc.) inside real world using technology of Augmented Reality. Possibility to connect additional informations with real world objects make Augmented Reality very usable in the field of education.

The evolution of modern web solutions during the past years has made the acquisition, recording and manipulation of virtual 3D data in Augmented Reality possible on contemporary web browsers. This technological advance open new possibilities for creation of AR educational materials without development of special smartphone application or use of special devices.

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Goal of this research is to present existing applications of Augmented Reality in different areas of IT education. Research analyze and compare different AR e-learning approaches for information technology education. With end goal to create set of recommendations and guidelines for future work on development of web based AR e-learning systems. Also, the posibility of using and integrating this technology in the field of interactive learning of Java and C and SQL will be considered.

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SESSION 5
CPMMI 2020

STOCHASTIC MODEL OF EPIDEMIC SPREAD

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In the represented work problems of the stochastic description as the determined mathematical models and stochastic are discussed. The original position is represented such: the solution of the determined task is mean value of its stochastic analog. A study of the development of epidemics based on the modified normal law of probability distribution was proposed.

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PREISACH MODEL FOR MILD STEEL HYSTERETIC BEHAVIOUR

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In this paper, the new type of Preisach model, that describes the elastoplastic behavior of structural mild steel under axial monotonic load, is developed. It is the multilinear mechanical model, that describes Type 4 material behavior, typical for structural mild steel with pronounced yield plateau. The numerical model and appropriate Preisach triangle,suitable for engineering practice, were defined. This model was verified by comparisonwith the experimental results. The experiment was carried out on structural steel S275, and S355 specimens, under monotonic axial load.

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MULTI-USER MIMO PROCESSING USING HYBRID CSI WITH LARGE ANTENNA ARRAYS

Veljko Stanković
State University of Novi Pazar, Serbia

In this paper we introduced a novel multi-user multiple-input multiple-output precoding algorithm in case when the number of antenna elemnts is much larger than the number of data streams that is transmitted to the user terminals. The drawback of massive multiple-input multiple-output is requirement for large pilot overhead for estimation of the channel state information. We use additional degrees of freedom due to the large number of antenna elements to design procoding matrices using hybrid channel state information. Multi-user interference is suppressed using long-term channel state information, and the performance optimization is perfomed using the instantaneous channel state information that does not require orthogonal pilots per user terminal. We aim to reduce the pilot overhead requirements, and approach performance limits of the spatial processing.

***

CREATING A STOP WORD DICTIONARY IN SERBIAN

Ulfeta Marovac, Aldina Avdić, Adela Ljajić
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In the era of IT expansion, text mining is very important in many fields because it can lead to new knowledge. By using text mining techniques, we get answers from search engines, companies based on our writings on social networks conclude about our needs and offer us products and services, etc. It is therefore of utmost importance to separate what carries informative value in a sentence from those words that do not affect its meaning and serve to make meaningful connections between significant words.

Removing stop words is one of the steps of normalizing textual data. In Serbian there is no stop word resource in electronic form suitable for further use, as well as a method for automatically extracting stop words from the corpus. Available sets of stop words are usually created by translating this resource from other languages and are incomplete in terms of number of stop words and their description.

This paper describes a domain-independent, Serbian-language stop word resource that can be used for various applications. Then a method for automatic extraction of stop words is proposed. The experimental section provides a comparison of the created resource with existing resources and the impact of stop word removal on document size, as well as the results of applying the proposed method for automatic extraction of stop words.

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AUTOMATIC LABELING OF DIAGNOSIS IN MEDICAL REPORTS IN SERBIAN

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A large number of patient health data is collected daily in medical information systems. This information contains a non-structural part written in natural language that contains the physician's notes on specific characteristics of the patient's medical condition. This section contains symptoms, diganoses, therapies, specialties, Latin terms, and other words specific to the medical domain. Useful information suitable for various analyzescould be extracted by processing this section of the text.

There are no electronic lexical resources in the Serbian language that are suitable for normalizing and extracting knowledge from medical texts, as well as methods for marking terms in this domain. One reason is that, before any method is applied, the de-identification of patients and staff must be ensured. Also, the

evaluation of the results requires manually marked corpora of medical reports in the Serbian language.

This paper proposes a method for identifying words belonging to diagnoses in medical texts written in Serbian using natural language processing (NLP) techniques. The proposed method is based on the use of lexical resources, and a set of 1000 medical reports is manually marked for research purposes. In the experimental part, the results of automatic labeling of diagnoses on the marked corpus using the proposed method are presented.

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INOVATIVE ENVIRONMENT FOR LEARNING OF THE BASIC OOP PRINCIPLES BASED ON VISUAL REPRESENTATION OF THE OF THE PROGRAMMING ALGORITHMS

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This research will cover usage of visual programing languages in covering issues and learning of Object oriented programming (OOP). We used behavioral programming principles applied on visual programming language blocky for implementation of OOP concepts in Phyton programming environment. This approach created innovative environment for learning of the basic OOP principles in Phyton, developed on visual representation of the of the programming algorithms.

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APPLICATION OF BSO ALGORITHM FOR ROBOT PATH PLANNING IN UNCERTAIN ENVIRONMENT

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Robots are capable of performing operations without the help of a man or with minimum human activity, usually performing tasks which are dangerous, stressful, hard, or boring for humans. Mobile robots are more useful then static ones, but there are more challenging problems that are related to them. One of the major problems of mobile robots is path planning, which refers to finding the optimal route from the start position to the target, where optimality can be defined in numerous ways. In real life situations, environment in which robots move may contain not only static or mobile obstacles, but also obstacles or danger sources with unknown or uncertain positions. Therefore, path planning problem is a multi-objective constrained NP-hard optimization problem, and many different approaches, both classical and heuristic, have been proposed to solve it. It has been shown that the heuristic approaches were very successful in providing good solutions, especially the metaheuristic

ones. Brain storm optimization algorithm (BSO) is new and very promising metaheuristic, swarm intelligence algorithm based on the collective behaviour of humans when solving a problem, i.e the brainstorming process.

In this paper we proposed the application of BSO algorithm for solving the path planning problem of mobile robot in complex environment which contains obstacles and stochastic danger sources. The probabilistic model was used to determine danger degree for sources with unknown exact positions. Performance criteria were path length (fuel consumption) and safety degree. Based on the simulation results with benchmark examples, it can be concluded that our proposed approach is, even though simpler, robust and more efficient since it exhibited better performance in all cases.

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ON THE PROBLEM OF HEAVY INVERTED COLUMN WITH CONCENTRATED FORCE AT THE END

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We consider stability problem in the gravitational field and the problem of optimal shape of an inverted elastic column with concentrated force at the end. Earlier results on this problem we generalize by proving the bifurcation theorem of nonlinear equilibrium equations for an arbitrary cross-section of a column. After that, we determine the optimal cross-sectional area of the compressed column.

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About Novi Pazar

The Conference will take place on the State university of Novi Pazar. The city of Novi Pazar is a significant economic, cultural and educational center in the southwest of Serbia and the city where the State University of Novi Pazar is situated. The location where it is placed is surrounded by the mountain Rogozna in the south and southeast, the river Ibar valley, mountain Golija in the north and northwest, Sjenica-Pester plateau in the west, and Mokra Gora and river Ibar valley, in the south.

The city is an area where various civilizations alternated from the prehistoric period to the present. It is particularly important its medieval heritage, since the first Serbian statehood and spirituality were born here, with cultural

and historical monuments: Monastery of the Holy Apostles Peter and Paul from the 9th and 10th century, the monastery The Pillars of Saint George from the 12th century, the Sopocani Monastery from the 13th century, the old town of Ras etc.

Novi Pazar is also rich in cultural and historical monuments from the Ottoman period, when the city was one of the most important and largest trade centers in the Balkans: the Altun Alem mosque, which was built between 1516 and 1518, the Lejlek Mosque, Arab Mosque, Turkish bath, Novi Pazar Fortress etc.

The Altun Alem   The Sopocani Monastery   Turkish bath in N.Pazar